

## ***Chapter 4 - Qualitative Analysis***

<b>4.1 Introduction .....</b>	<b>3</b>
4.1.1 Background.....	3
4.1.2 Reading Material .....	4
4.1.3 Software.....	5
<b>4.2 Qualitative Analysis Techniques.....</b>	<b>6</b>
4.2.1 Introduction.....	6
4.2.2 Conditions of Structural Behaviour.....	7
4.2.3 Methods to Aid Solution.....	14
4.2.4 Example 1 .....	15
4.2.5 Example 2 .....	20
4.2.6 Example 3 .....	22
4.2.7 Example 4 .....	24
4.2.8 Example 5 .....	26
4.2.9 Example 6 .....	28
4.2.10 Example 7 .....	32
4.2.11 Example 8.....	34
4.2.12 Example 9 .....	37
4.2.13 Example 10.....	39
4.2.14 Example 11 .....	42
<b>4.3 Problems.....</b>	<b>47</b>
4.3.1 Introduction.....	47
4.3.2 Statically Determinate Beams .....	48

4.3.3	Statically Determinate Frames.....	50
4.3.4	Statically Indeterminate Beams .....	56
4.3.5	Statically Indeterminate Frames .....	58
4.3.6	Trusses .....	64

Rev. 1

## 4.1 Introduction

### 4.1.1 *Background*

The ability to ‘see’ and interpret structural behaviour is a core ability of a structural engineer. At the initial stage of a structural scheme design, we are not interested in numbers, or amounts, only the sense of a load effect. Some examples of what we mean by sense are:

- Is there tension on the top or bottom of a beam?
- Does the tip of a cantilever deflect up or down?
- Is the moment reaction clockwise or anti-clockwise?

Getting this level of analysis right is not only the first step, but the most important step. If we don’t get this level right, then the answers to a more complicated analysis will be meaningless.

The ability to get the right answers to this level is called *Structural Intuition*. The better your structural intuition, the better you will be a designer. This ability reduces errors both in design practice but also whilst in college: since you will already ‘see’ the answer it is easier to catch errors in calculations.

### 4.1.2 Reading Material

Some good books on structural behaviour are:

- Brohn, D., *Understanding Structural Analysis*, 4th Edn., New Paradigm Solutions, 2005.
- Jennings, A., *Structures: from theory to practice*, Spon Press, 2004.
- Ji, T., and Bell, A., *Seeing and Touching Structural Concepts*, Taylor & Francis, 2008.
- Hilson, B., *Basic Structural Behaviour: Understanding Structures from Models*, Thomas Telford, 1993.
- Pippard, A.J.S., *The Experimental Study of Structures*, Edward Arnold & Co., London, 1947.
- I.Struct.E., *Qualitative Analysis of Structures*, London, 1989.

Due to its importance, the Ove Arup Foundation sponsored the report: *The Teaching of Structural Analysis* by Prof. Ian May and Dr. David Johnson. It is accessible here: <http://www.jbm.org.uk/uploads/StructuralAnalysiswithCover.pdf>.

A summarized version of the report appeared in *The Structural Engineer*, Vol.81, No.7, 2003, p.33-37, available at this link:

<http://www.istructe.org/thestructuralengineer/Abstract.asp?PID=7904>

### 4.1.3 Software

In developing your structural intuition, it is very helpful to model structures using an appropriate computer program – especially when the structure behaves counter-intuitively. Most structural analysis programs today are extremely complex with many options and capabilities and this can often obscure the modelling process.

An appropriate program (for a few reasons) is *LinPro* – freely available from [www.line.co.ba](http://www.line.co.ba). You should install *LinPro* on your own computer. Also, it is installed on the computers in Rm 392.

The program is intuitive to use and comes with a reasonable help file. If you have any difficulties using the program, please ask the lecturer.

Another program for the analysis of trusses is *TrussMaster*, developed by the lecturer for the purposes of teaching structural behaviour of trusses. This is available on the college computers, and a User Manual is also available at [www.colincaprani.com](http://www.colincaprani.com).

## 4.2 Qualitative Analysis Techniques

### 4.2.1 Introduction






Qualitative Analysis is not a linear process. For some problems we might start with reactions and proceed through bending moments to a displaced shape, whilst for others we may begin with a displaced shape, work out reactions, and then find the bending moment diagram. The approach to use will depend on the problem and there are set rules or procedures that you can follow to be guaranteed to arrive at the correct solution.

On a more positive note, since the structure will only behave in one distinct manner, there can only be one correct solution. By definition therefore, incorrect solutions will contain inherent incompatibilities. For example, all aspects of a frame's solution may agree (e.g. reactions, bending moment diagram, etc.), but it may require a rigid joint to have different rotations. Since this is impossible, we know that this cannot be the right answer. Therefore if we have a proposed solution, we must ensure that it does not violate any of the conditions of structural behaviour. If it does, then it is not the correct solution. In other words, your answer will tell you if it is wrong or not!

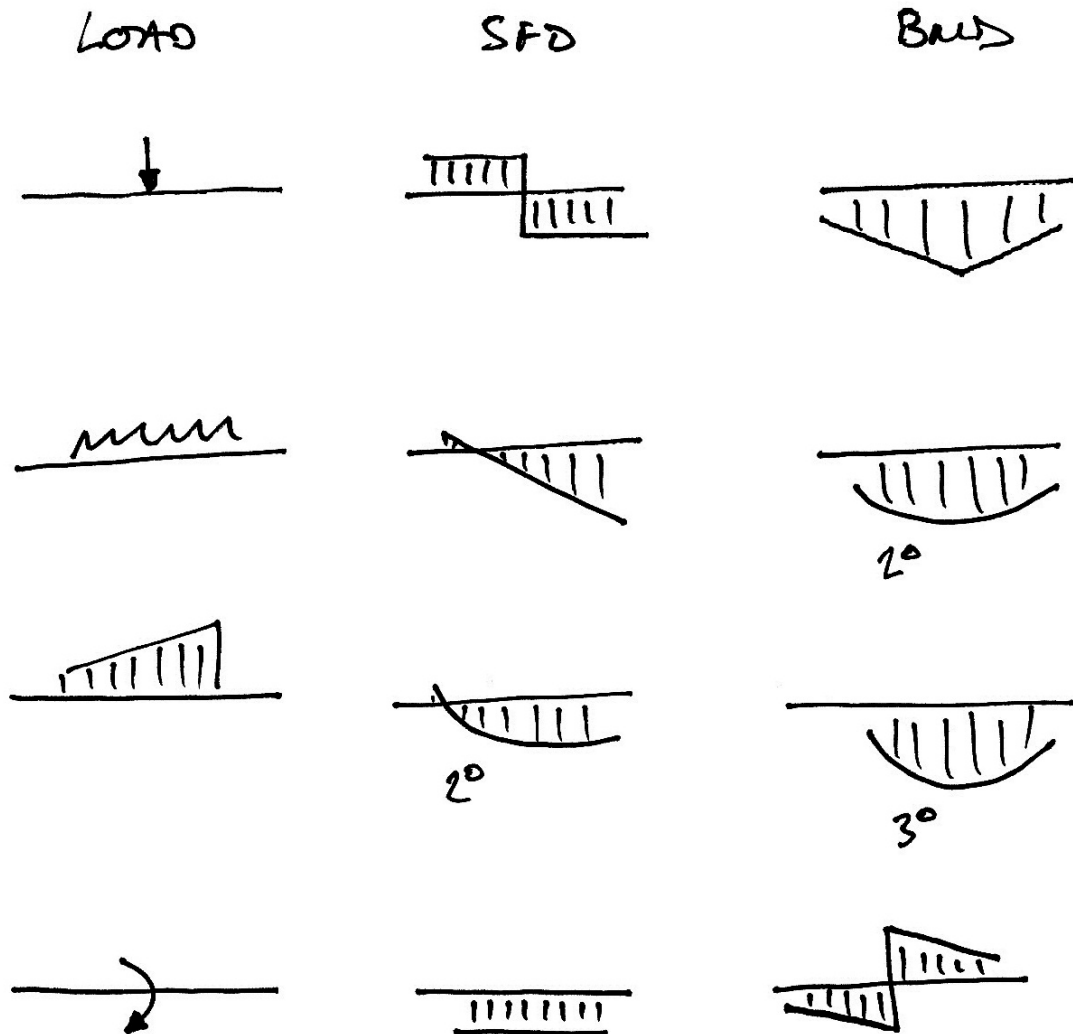
### 4.2.2 Conditions of Structural Behaviour

There are certainties about structural behaviour that we can rely on when attempting to analyse a structure. Most of these are plainly obvious, but a few may not be.

1. Remember the very basics: moment = force  $\times$  distance.
2. Know your support types and the type of restraint they offer:

Symbol	Name	Movements
	Roller (Horizontal)	$\delta_x$ <del><math>\delta_y</math></del> $\theta$
	Vertical Roller	<del><math>\delta_x</math></del> $\delta_y$ $\theta$
	Pin	<del><math>\delta_x</math></del> <del><math>\delta_y</math></del> $\theta$
	Fixed	<del><math>\delta_x</math></del> <del><math>\delta_y</math></del> <del><math>\theta</math></del>
	Vertical Support (beam continuous over the support and can rotate)	$\delta_x$ <del><math>\delta_y</math></del> $\theta$

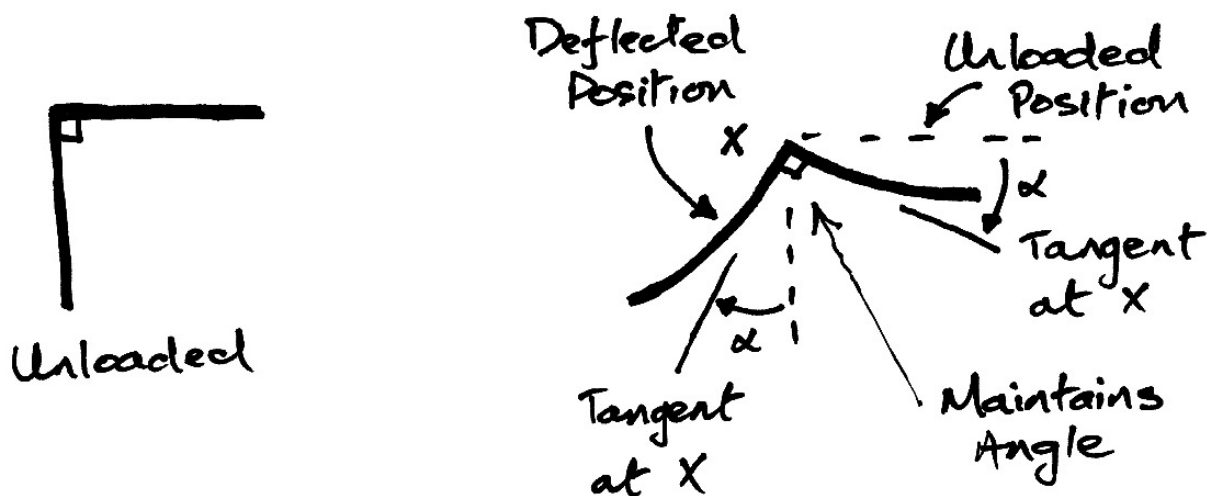
3. Recall the shapes of BMD and SFD under the different types of loading (rectangular, triangular, parabolic).



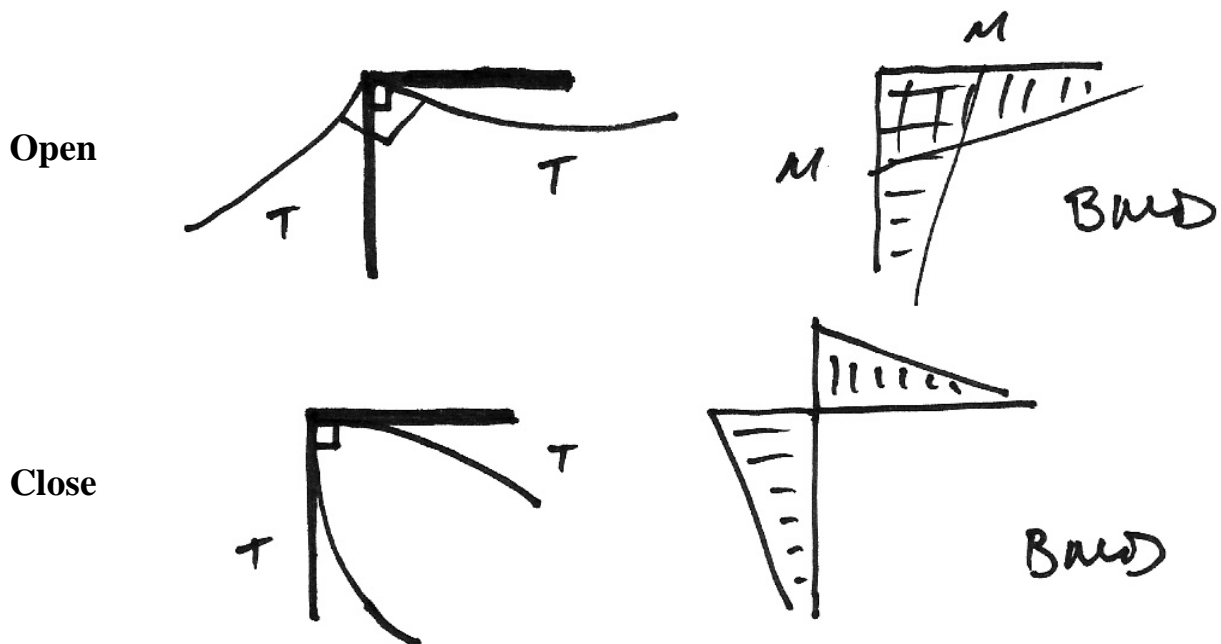
4. Remember: shear is rate of change of moment.
5. No transverse load or end shear force on a frame member means there is constant BM along the member (constant may equal zero).
6. There is zero bending moment at a hinge.



7. Members with no bending moments remain straight (i.e. no bending), but may still move.
8. Always draw the bending moment diagram on the tension face of the member. to be consistent with our convention
9. Remember, fixed supports will have a moment reaction, pinned supports will not, though there may be an external moment applied at a pinned support.
10. For unbraced frames, only symmetrical such frames symmetrically loaded will not sway.
11. Keep in mind: deflections are always small and we neglect the self weight of the structures – only analyse for the loads shown.
12. Deflected shapes are always very smooth curves, except at a hinge.
13. Rigid joints in frames must keep the same angle as they rotate.

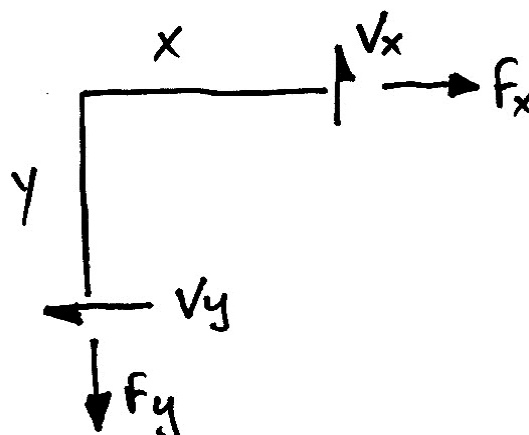


14. Rigid joints can only open or close:

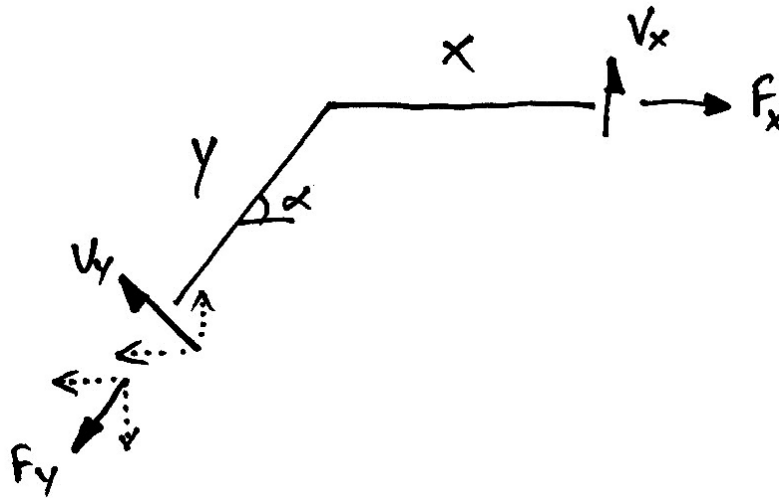


15. At a rigid joint with two members, there is only one value of moment ( $M$  above). (There is one rare exception to this rule)

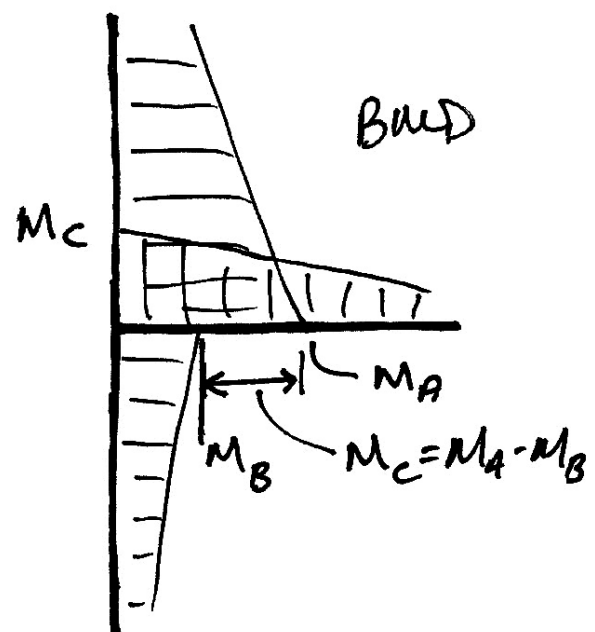
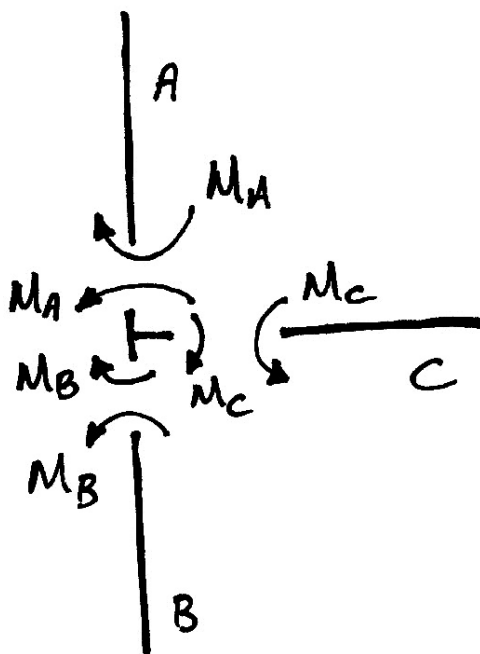
16. At a right-angle rigid joint, the shear becomes the axial and the axial becomes the shear in the alternate members. Just use  $\sum F_x = 0$  and  $\sum F_y = 0$  to see why.



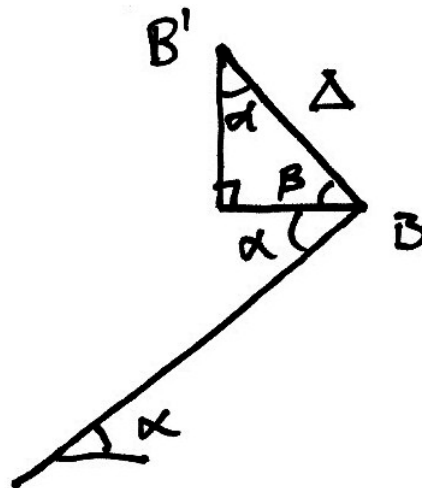
17. This is not the case for oblique-angle joints:



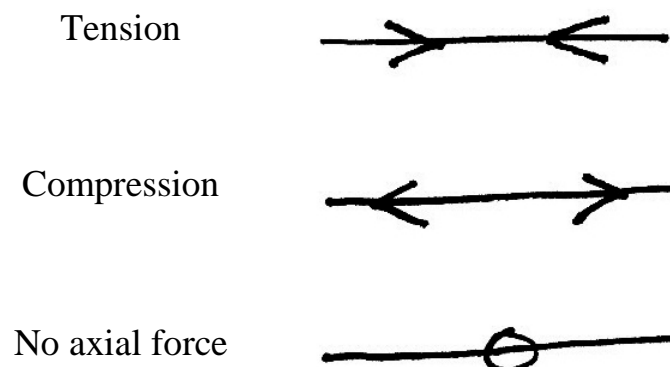
18. When more than two members meet at a rigid joint, the joint must be in equilibrium. This means that for the joint below,  $M_A = M_B + M_C$ . A further implication of this is seen in the BMD: there is a step in the bending moment for member AB at the joint of value  $M_C = M_A - M_B$ .



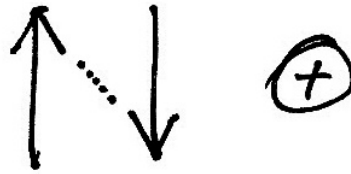
19. For frames, we normally neglect axial deformation. This means that members cannot change length and because deflections are small, this means that the member's joints must move perpendicular to the line of the member. For example, below  $B$  can only move along the line  $BB'$ .



20. Trusses do not have bending moment diagrams.
21. Remember the axial force sign convention:



22. Positive shear force sign convention makes the letter 'N': up on the left, down on the right:



### 4.2.3 *Methods to Aid Solution*

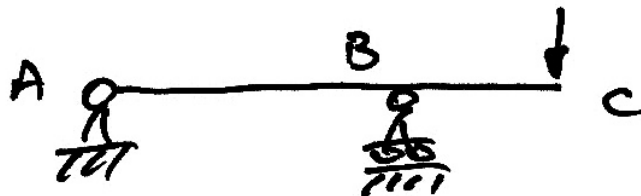
The following are some methods that may help you carry out the analyses:

1. To find a support reaction, *Remove the Restraint* offered by the reaction and draw the deflected shape of the resulting structure. Apply the support reaction in such to as to bring the structure back to where it should be.
2. Use *Points of Certainty* – where you know the deflected position, for example at a support the deflection is zero, and usually the structure moves away from the applied load (though there are rare exceptions).
3. For more complex structures, remove excess members/supports/joints and reintroduce one at a time and observe the effect each additional feature has.

### 4.2.4 Example 1

#### Problem

Analyse the following beam for the reactions, bending moment and shear force diagrams, and draw the deflected shape.

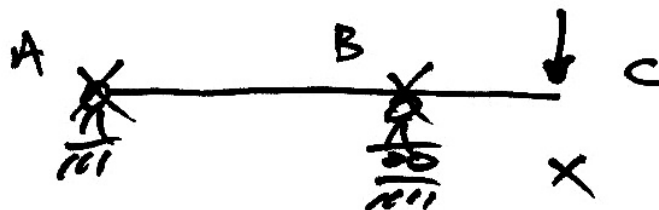


#### Solution

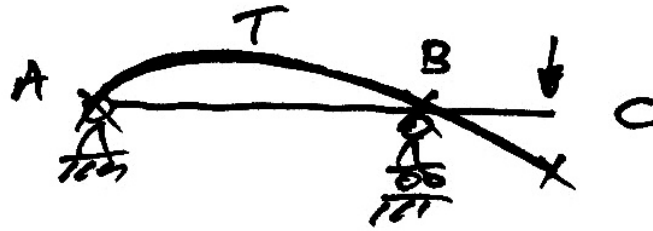
Firstly we identify the *Points of Certainty*:

- It cannot move horizontally or vertically at A;
- It cannot move vertically at B;
- It will probably move downwards at C away from the load.

This gives the following points through which the deflected shape must pass:



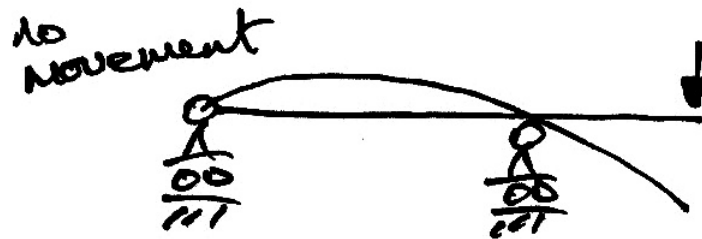
Noting that the deflected shape is always a smooth curve (except at hinges, of which there are none here), we join the three points with a smooth curve:



Also, we know there is tension (“T” above) on the outside of the curve and so we include this in our drawing. This helps inform us of the bending moment diagram – always draw it on the tension face.

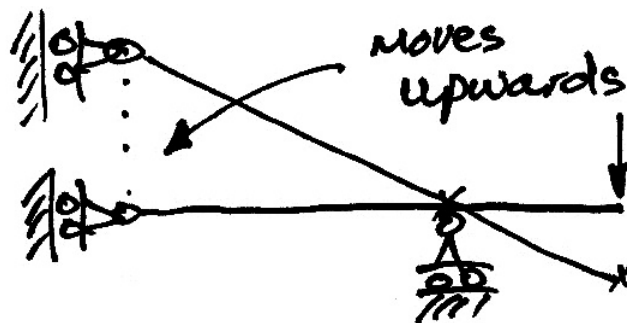
To find the direction of the reactions, we will remove each restraint in turn and follow the above steps to see how the beam deflects when the restraint is removed.

- For  $H_A$ :



Since there is no movement of the beam when  $H_A$  is released,  $H_A = 0$ .

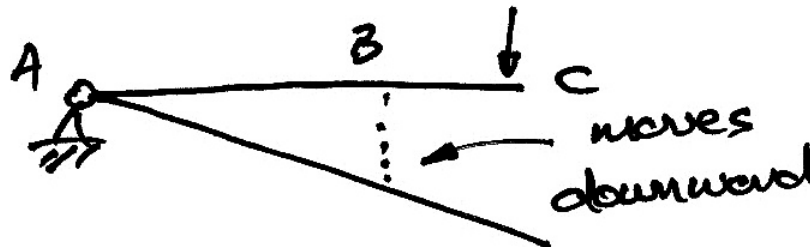
- For  $V_A$ :





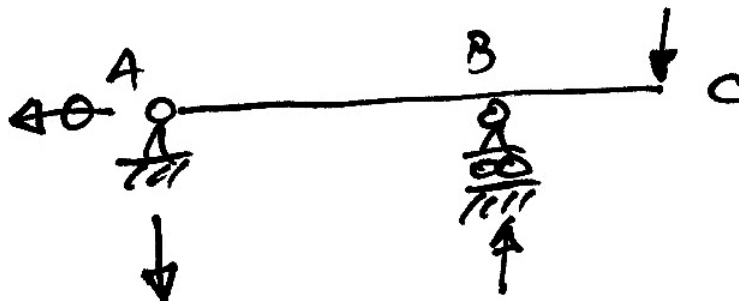
Since when  $V_A$  is removed it moves upwards, it must be that  $V_A$  acts downwards in the actual structure to keep the beam at  $A$  where it must remain.

- For  $V_B$ :

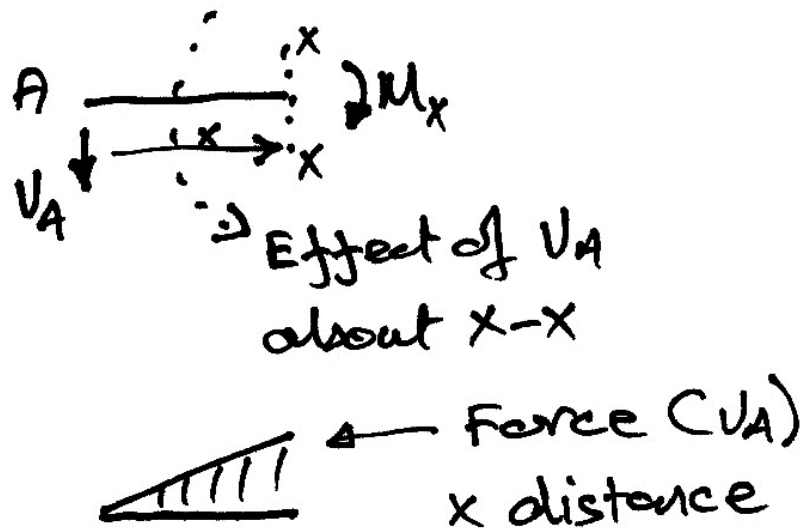


Since when  $V_B$  is removed the beam moves downwards,  $V_B$  acts upwards in the actual structure to ensure that  $B$  remains where it should.

Thus the reactions are:

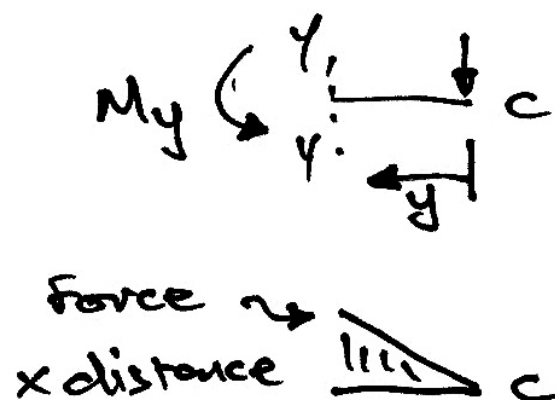


With this information it now becomes easier to establish the bending moment and shear force diagrams. Starting with the bending moment diagram for the portion  $AB$  of the beam, we take a cut somewhere to the right of  $A$ :

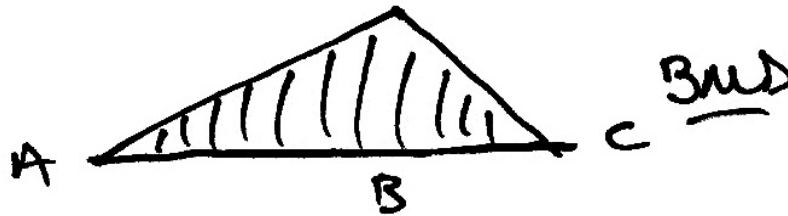


As may be seen, the effect of  $V_A$  is to cause an anti-clockwise rotation of the segment which must therefore be resisted by a clockwise internal bending moment  $M_X$ , as shown. This means (since the arrow comes from the tension face) that tension is on the top of the beam, and is increasing as the distance increases (the force remaining constant).

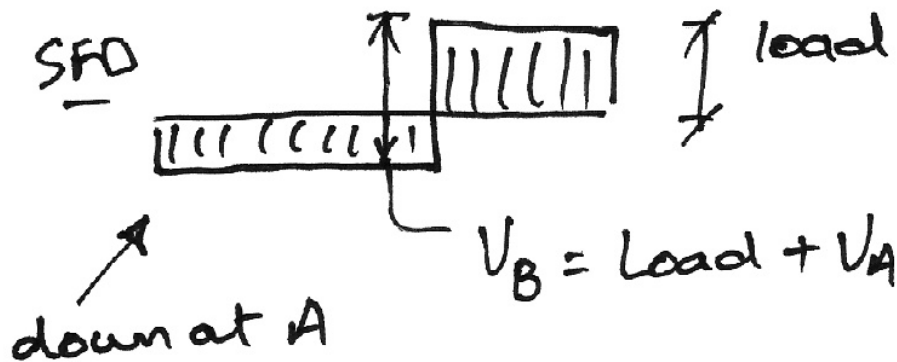
Similarly, we examine the portion BC:



There are no applied moments, so the bending moment diagram does not have any steps in it. This means that the two portions that we have identified above must meet over  $B$  to give:



The shear force diagram is easy to construct by just following the forces: moving left to right it is down at  $A$  then up at  $B$  over the line to a height equal to the applied force. The total height at  $B$  is the vertical reaction at  $B$  and this must sum to the total downward forces at  $A$  and  $C$ :

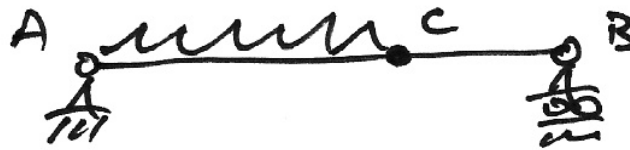


Note also that since  $V = dM/dx$ , we see that the negative shear corresponds to a negative slope in the BMD

### 4.2.5 Example 2

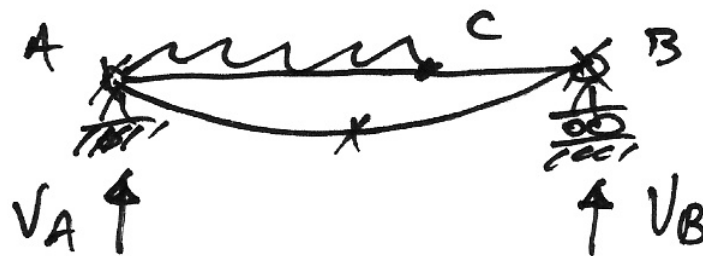
#### Problem

Analyse the following beam for the reactions, bending moment and shear force diagrams, and draw the deflected shape.



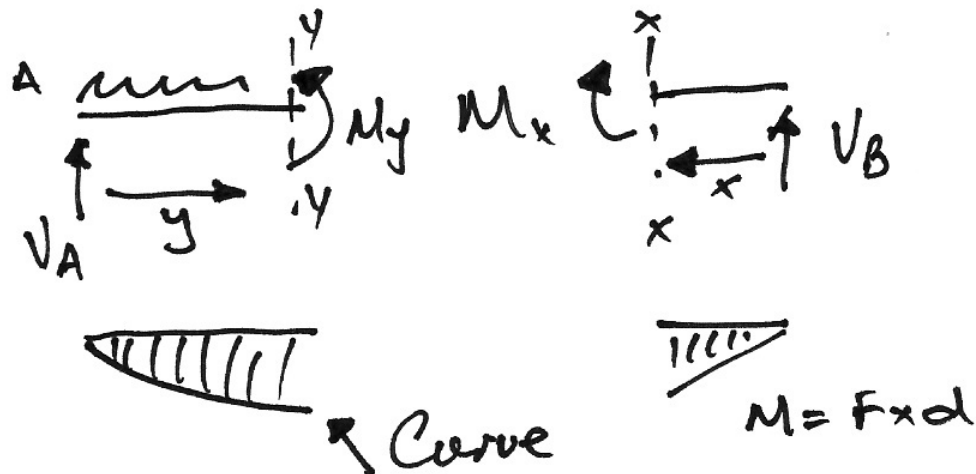
#### Solution

Using the same techniques as outlined in Example 1, we can quickly arrive at the deflected shape and reactions:

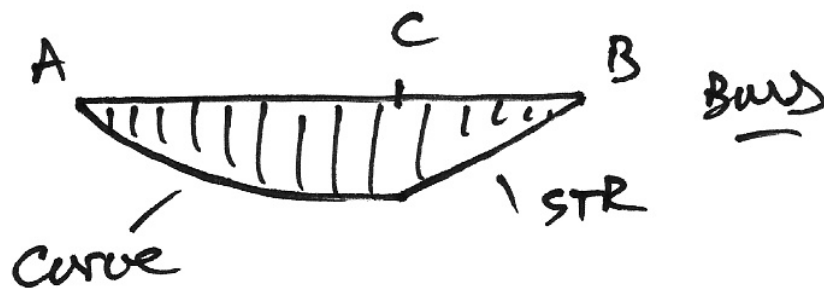


Based on the reactions, we can then examine the two portions of the structure for bending moments:

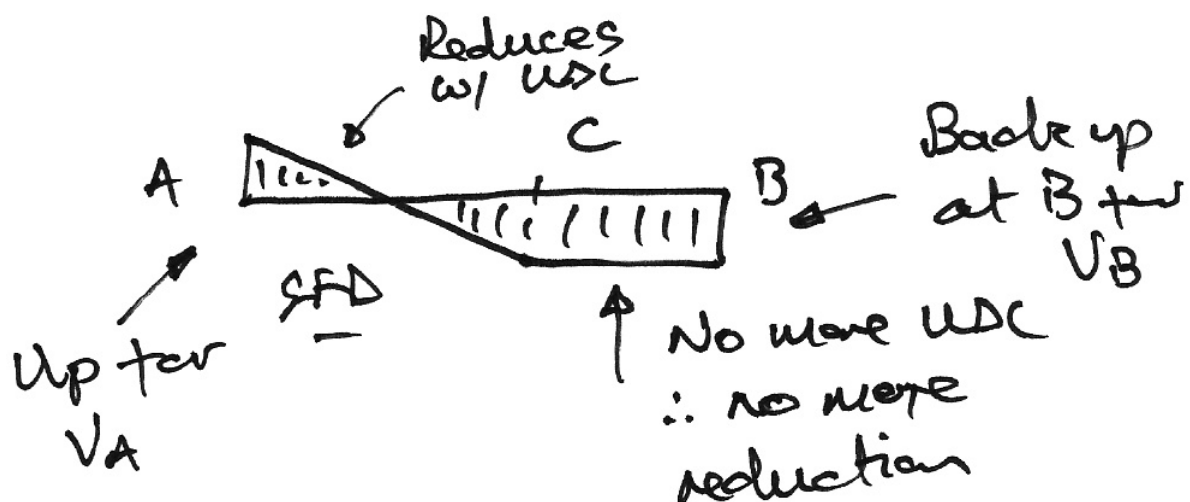
- **AC:** For this portion, the moment comes mainly from the reaction  $V_A$ . However, the moment gets progressively smaller than it would have been if just  $V_A$  was acting (i.e. force  $\times$  distance) since the UDL acts in the opposite direction. This means the BMD curves as shown below.
- **CB:** This portion is as studied in Example 1 is found from force  $\times$  distance.



Again, just like in Example 1, we recognize that we have no steps in the BMD and so join the moment diagrams for the two portions at C to get:



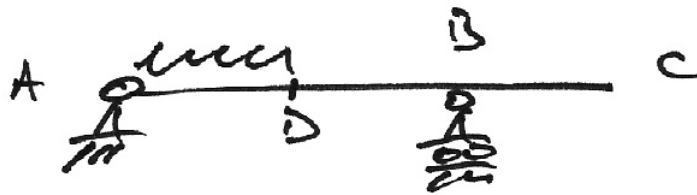
The shear force diagram is explained in the diagram:



### 4.2.6 Example 3

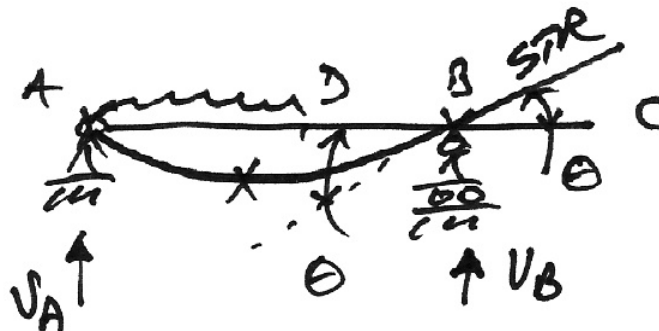
#### Problem

Analyse the following beam for the reactions, bending moment and shear force diagrams, and draw the deflected shape.



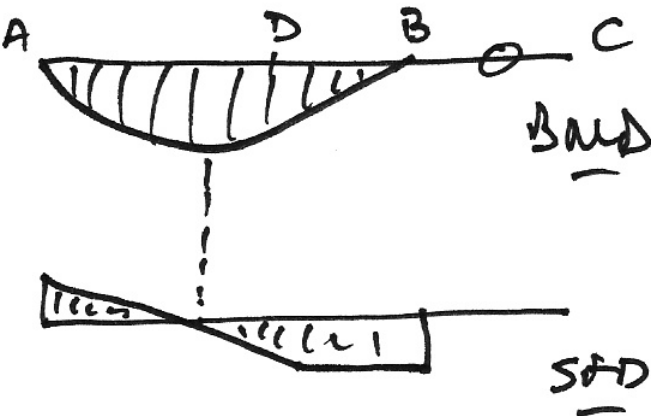
#### Solution

Again applying the techniques of Example 1 give the following deflected shape and reactions:



Note that for portion  $BC$  we recognize that there is no bending of the member. However, this does not mean that the member does not move: it does, and keeps a straight line extending the tangent to the deflected curve just to the left of  $B$ .

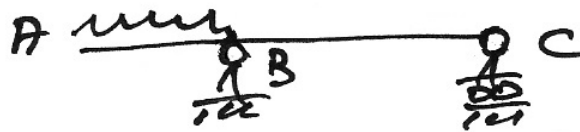
The bending moment and shear force diagrams are then found as per Example 2:



### 4.2.7 Example 4

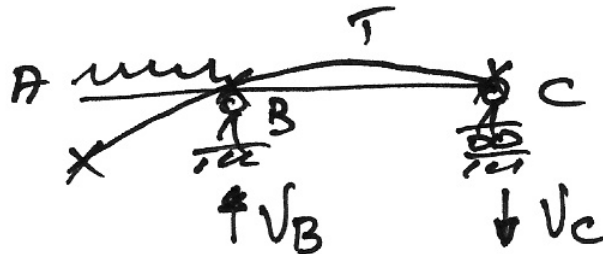
#### Problem

Analyse the following beam for the reactions, bending moment and shear force diagrams, and draw the deflected shape.



#### Solution

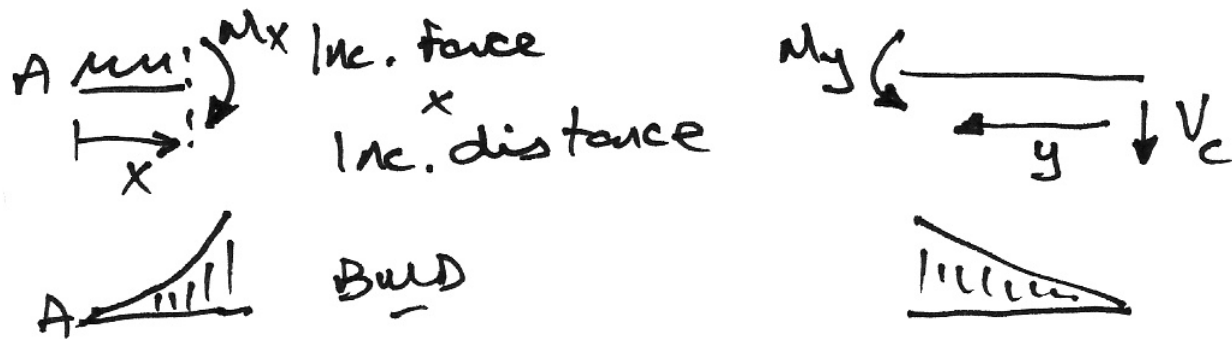
Again using points of certainty and removal of restraints we arrive at:



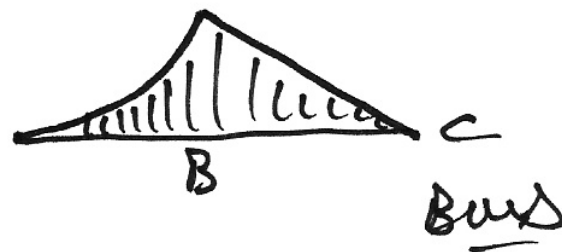
This allows us to look at the two portions of the structure for the BMD:

- *AB*: For this portion, we recognize that we have an increasing force (due to the accumulation of load from the UDL) as the distance increases. Thus we have a doubly increasing moment as the distance changes and so the BMD curves upwards as shown.
- *BC*: For this section, just apply force  $\times$  distance.

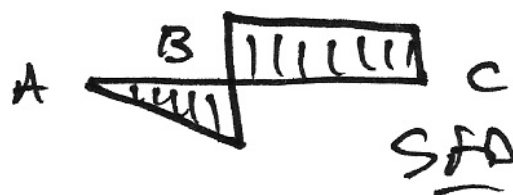




The BMD must not step (no applied moment) and so joins at B to yield:



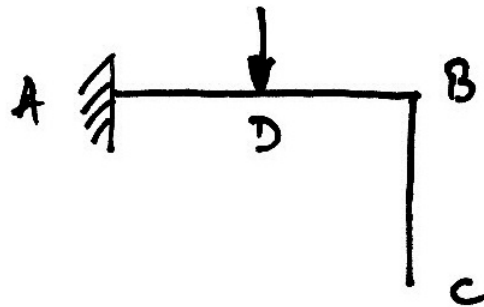
And the shear force diagram follows, either from the load types, or by looking at the BMD (curve to slope, line to constant).



### 4.2.8 Example 5

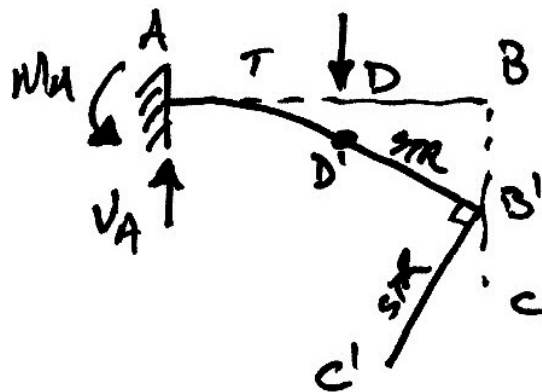
#### Problem

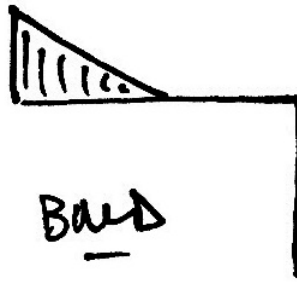
Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.



#### Solution

For this frame, we will start by establishing the reactions. First, since there is no

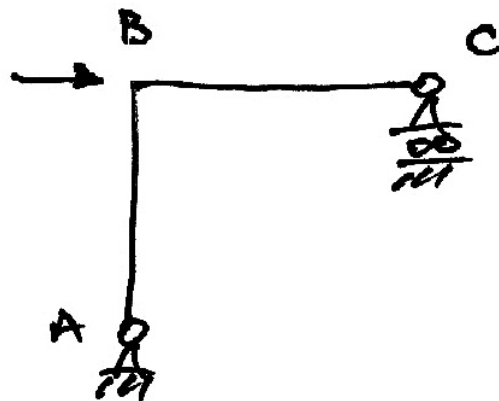




### 4.2.9 Example 6

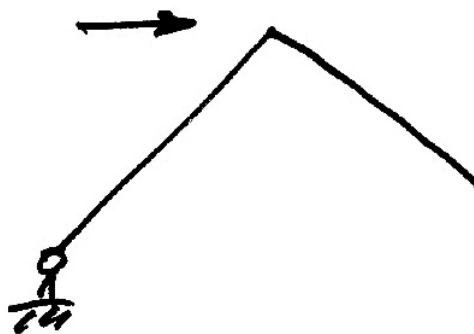
#### Problem

Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.

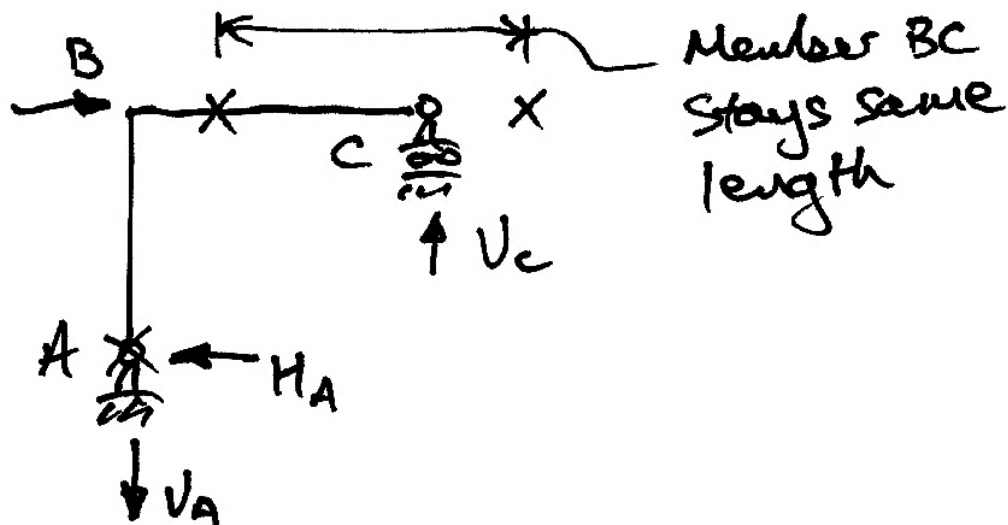


#### Solution

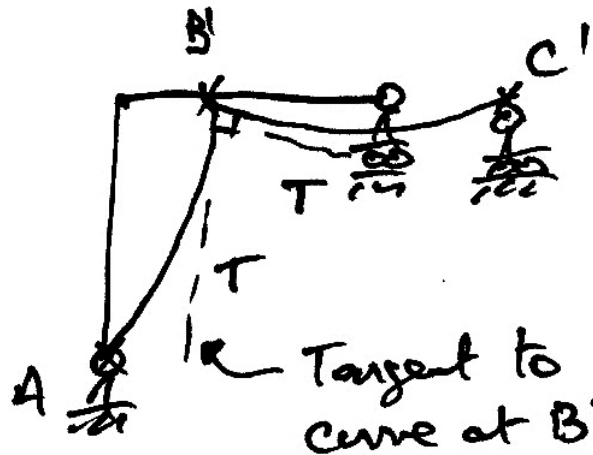
To begin, we will determine the direction of the vertical reaction at C. As ever, to do this, we will remove the restraint and examine what happens to the structure under the applied load:



Given this deflected shape, it is obvious that the vertical reaction at  $C$  should be upwards to keep  $C$  at the correct height. Since there are no vertical loads, this means that because of  $\sum F_Y = 0$ , we must have  $V_A$  acting downwards. Since there is no other possible horizontal force, by  $\sum F_X = 0$  we have  $H_A$  acting to the left. Thus we have:

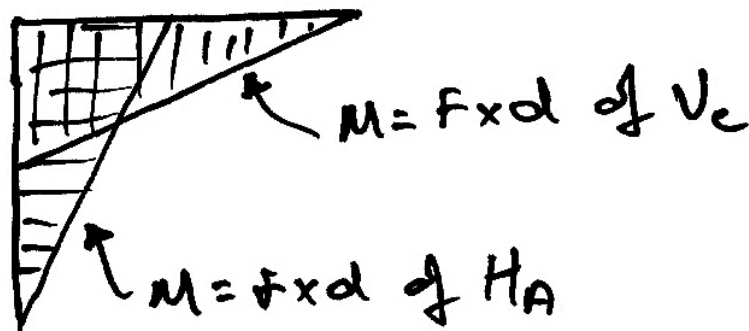


In the above diagram we have also indicated some points of certainty. That of  $A$  is easy due to the support. However, at  $B$  we note that the frame should move away from the load, but cannot move vertically downwards since member  $AB$  does not change length (ignoring axial deformation). This locates the deflected position of joint  $B$ . And, as indicated in the diagram, once the deflected location of joint  $B$  is known, so is that of joint  $C$ , because we know that member  $BC$  does not change length. Finally then, to assist us drawing the deflected shape between these points of certainty, we recognize that the joint is opening and so is rotating clockwise to give:

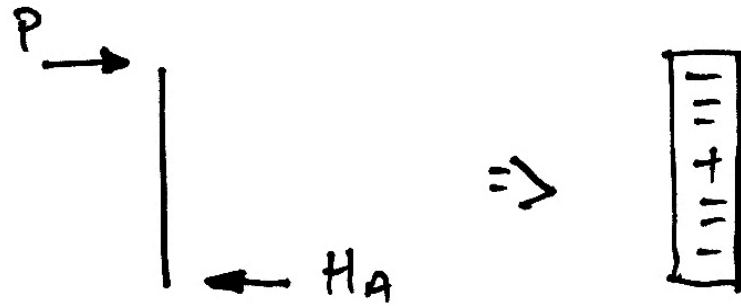


In the above, the tangents to the deflected shape curves are shown at joint  $B$  to demonstrate that the joint is rotating, but keeping its angle the same at  $B$ .

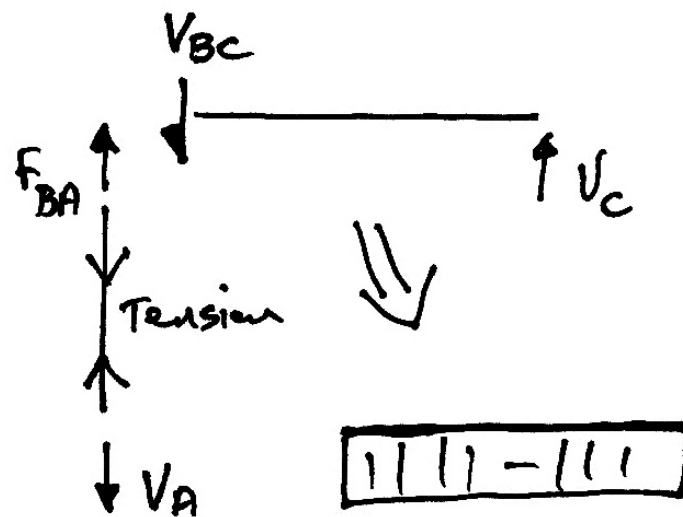
The BMD is easily established considering free-body diagrams of each member, along with the simple force  $\times$  distance:



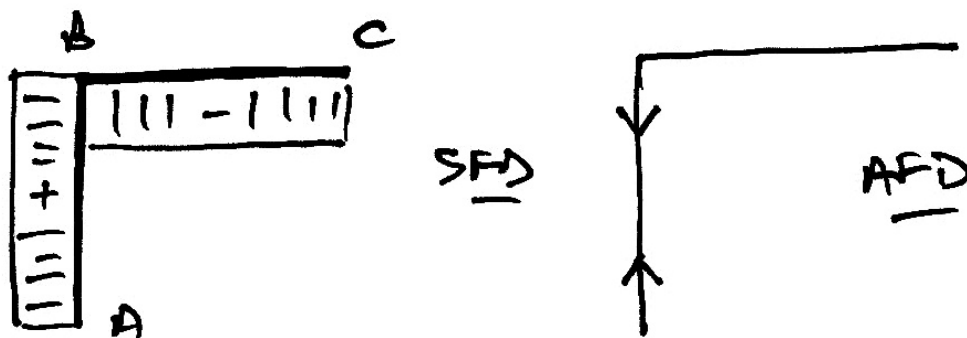
Recalling that shear is transverse forces to the member line, in considering the shear of member  $AB$ , we need only consider the applied load and  $H_A$  to get:



For the shear in member  $BC$ , we must first consider that the transverse force (besides  $V_C$ ) gets there through member  $AB$  as an axial force (caused by  $V_A$ ), to get:



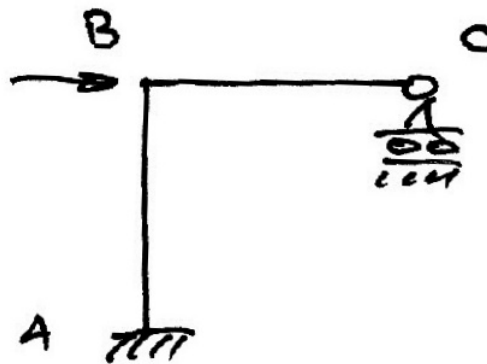
We combine the two solutions above to get the final shear and axial force diagrams:



### 4.2.10 Example 7

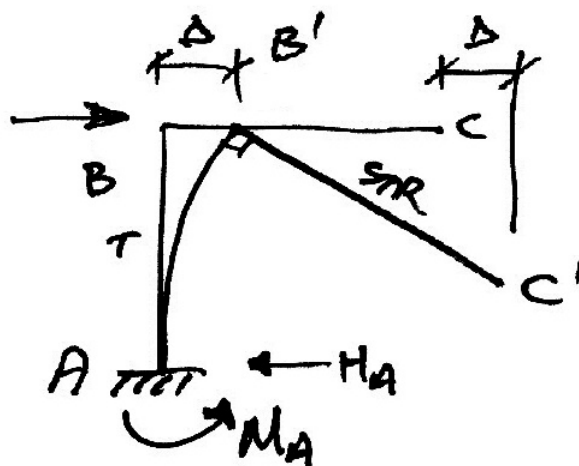
#### Problem

Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.



#### Solution

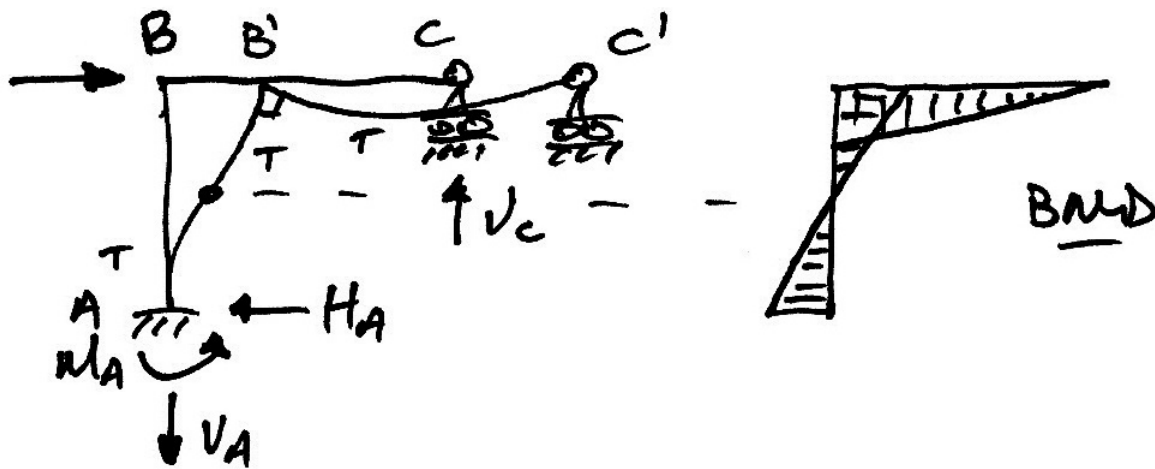
This frame is the same as that of Example 6, except for the support type at A. Thus we will see the influence of fixing a support on a structure. Firstly, proceed as we did before, and remove the support at C to get:





Notice that the diagram emphasises that the horizontal displacements at  $B$  and  $C$  must be the same since member  $BC$  does not change length. Also we see that we develop an anti-clockwise moment reaction at  $A$ .

Next, introduce the vertical support at  $C$ , noting that we now have an upwards vertical reaction at  $C$ , and proceed as before to get:



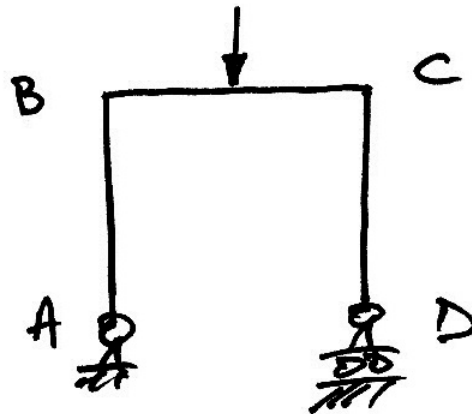
Notice that the *Point of Contraflexure* is noted as a dot in the deflected shape drawing, and its location is produced across to locate the zero point of bending moment on the column  $AB$ .

The shear force and axial force diagrams are obtained as was done in Example 6:

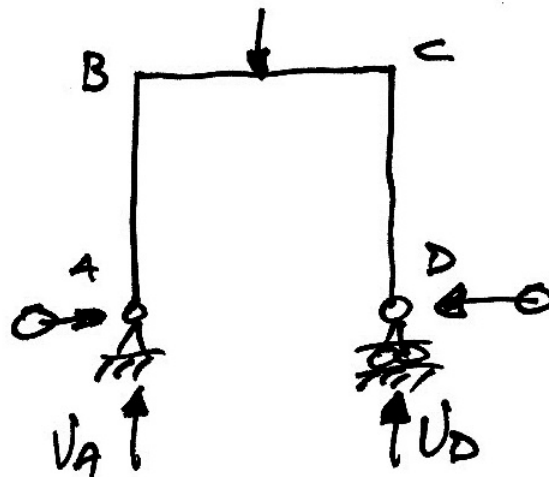


**4.2.11 Example 8****Problem**

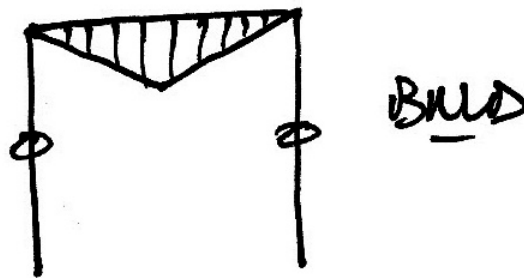
Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.

**Solution**

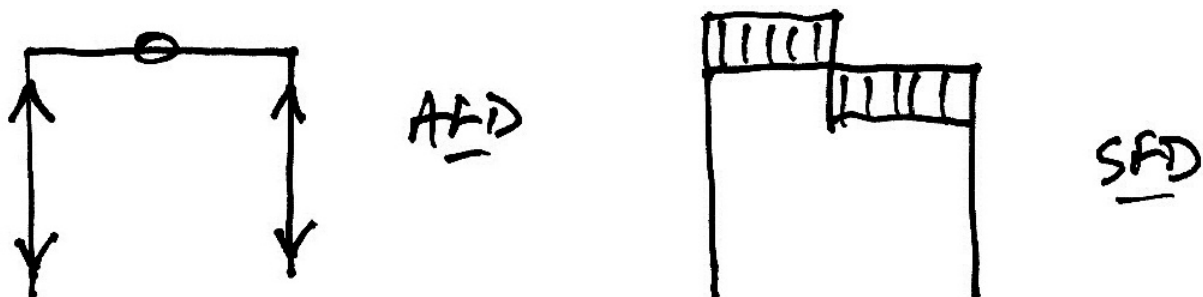
For this frame, we will start by establishing the reactions. First, since there is no horizontal support at  $D$ , and since  $\sum F_x = 0$ , we know  $H_A = 0$ . Also by considering removal of restraints we will see that the two vertical reactions are upwards, to give:



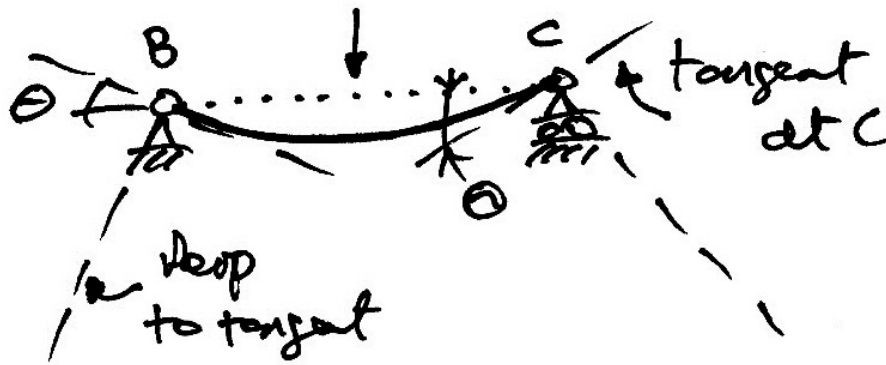
Bending moments are only caused by forces transverse to a member. Thus, with no horizontal reactions (i.e. no forces transverse to members  $AB$  or  $CD$ ), there can be no bending moments in the columns. This only leaves the beam  $BC$  to act as a simply supported beam, giving the BMD as:



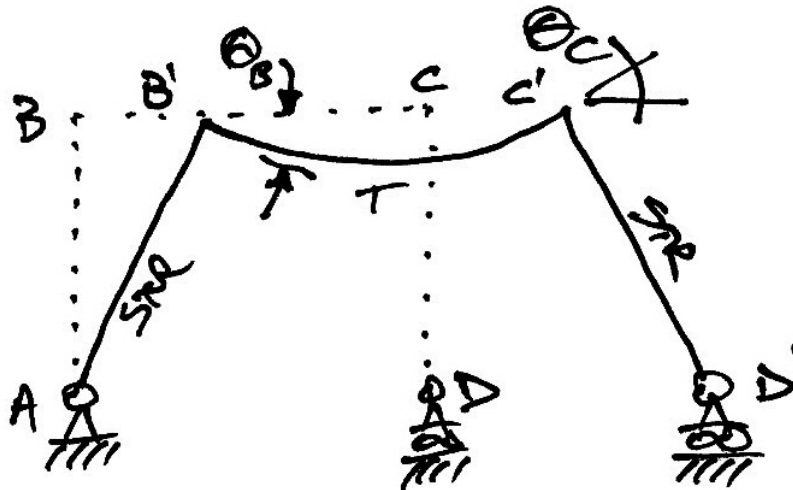
Next we note that the columns are in compression (by the reactions) and transmit the end shears of member  $BC$  to ground, whilst there is no axial force in the beam since there are no horizontal forces. The shear force and axial force diagrams are thus:



Lastly, we come to draw the deflected shape of this frame. However, before we do so, we recall that there is no bending in the columns and that member  $BC$  behaves as if a simply-supported beam. We examine the bending of member  $BC$  in more detail:



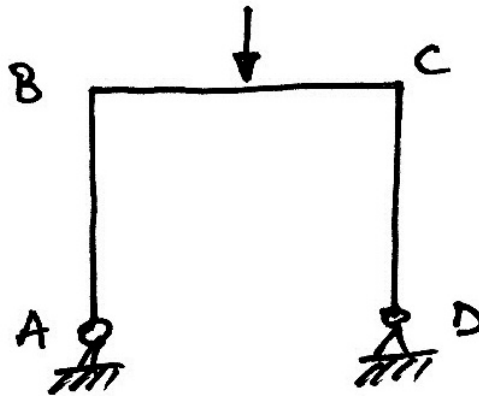
In this diagram we have identified the tangents to the end rotations of beam  $BC$  and the perpendiculars to these tangents. We recall that the right-angle rigid joints of the frame remain at right-angles, and so joints  $B$  and  $C$  of the frame rotate through  $\theta$ . However, since there is no bending in member  $AB$ , and since  $A$  cannot move (pin support),  $B$  must move to  $B'$  so that the rotation  $\theta$  can occur at  $B$ . Joint  $C$  and member  $CD$  behave similarly. Finally, we note that the distance  $BB'$  and  $CC'$  must be the same since member  $BC$  does not change length. All of this gives:



A way to think about it is that the frame sways to the right in order to avoid bending the columns.

**4.2.12 Example 9****Problem**

Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.

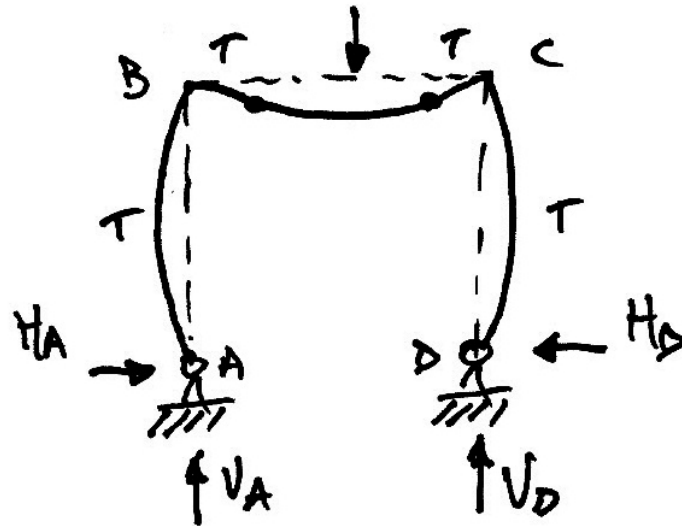
**Solution**

This frame is quite similar to the previous frame, except that  $D$  cannot move horizontally. This being the case, we must have a horizontal reaction acting to the left at  $D$ . Further, since  $\sum F_x = 0$  we must have a horizontal reaction at  $A$  opposing  $H_D$ .

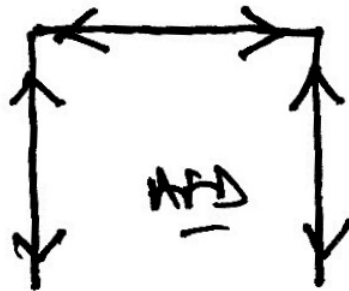
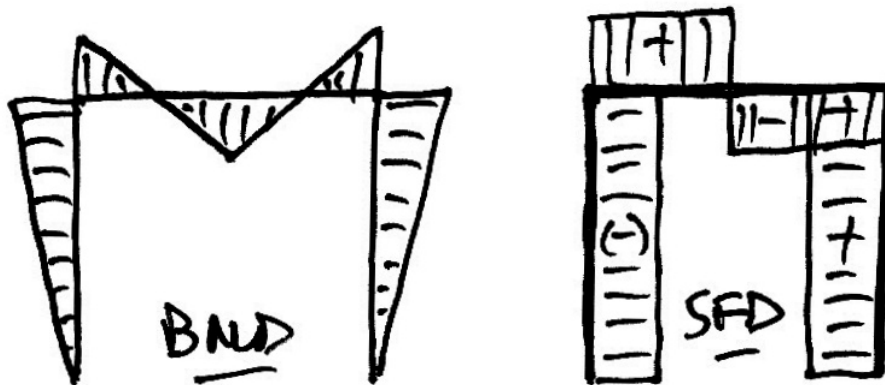
Please note this as it is a common misconception:

*Just because there are no applied horizontal forces, does not mean there cannot be any horizontal reactions (but if there are, they must balance).*

Finally for the reactions then, we note that the vertical supports must offer upwards reactions. Thus our deflected shape and reactions are:

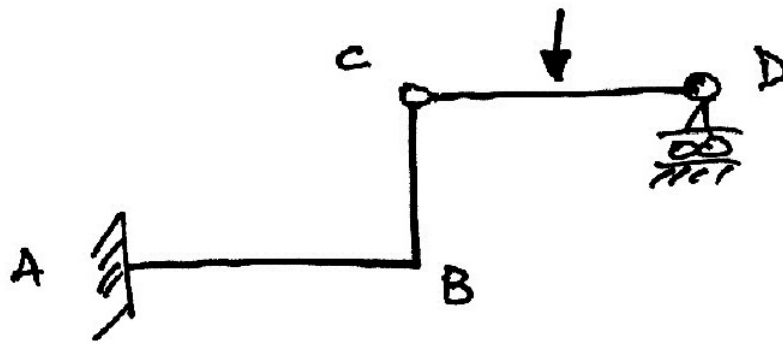


The BMD, SFD, and AFD follow directly by applying the techniques covered earlier given the reactions. Note especially that joints  $B$  and  $C$  are effectively closing and that beam  $BC$  behaves similar to a fixed-fixed beam:

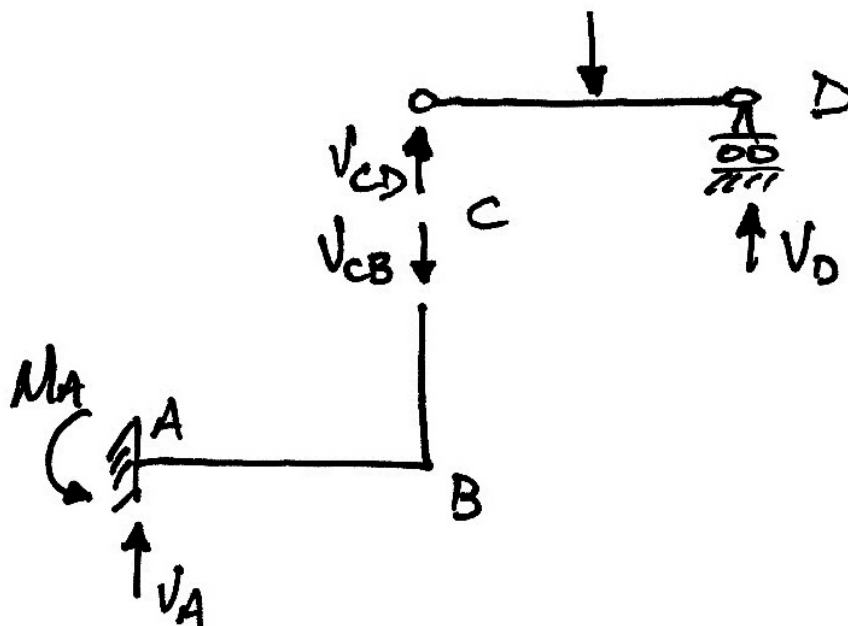


**4.2.13 Example 10****Problem**

Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.

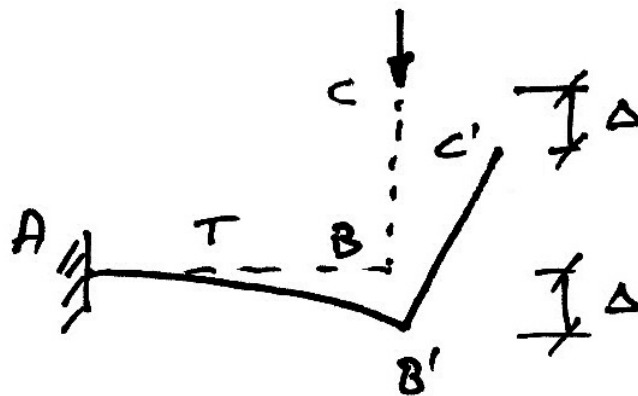
**Solution**

To proceed with this frame we will split it at the hinge:

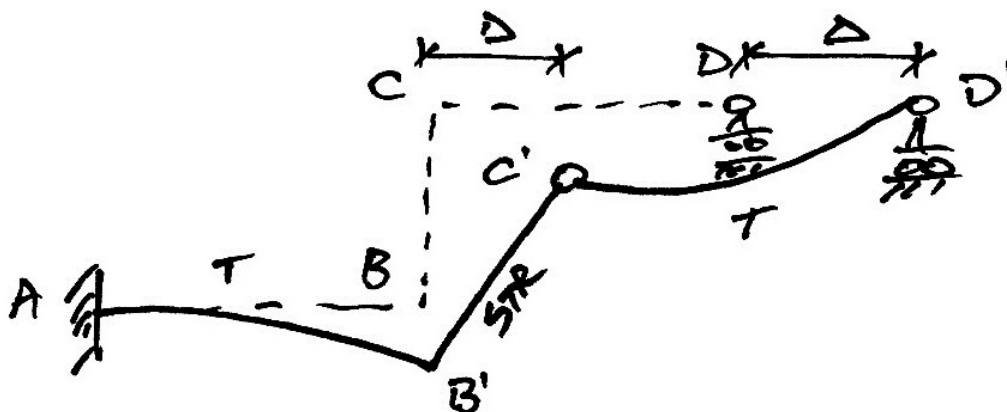


We note then that  $CD$  is effectively a simply supported beam, and this gives the interaction force direction as upwards for  $CD$  (reflecting the support that the structure  $ABC$  offers) and downwards for  $ABC$  (reflecting the push coming from the load on the beam). From  $V_{CB}$  we can determine the moment and vertical reactions at  $A$ .

The deflection behaviour of the beam  $CD$  is straightforward. We examine the deflection behaviour of  $ABC$  noting that  $B$  moves away from the load (downwards) and member  $BC$  maintains the perpendicular angle to the tangent at  $B$ . Moreover, member  $BC$  has no transverse force as so remains straight (i.e. does not bend). Lastly, see that the vertical movement of joints  $B$  and  $C$  must be the same since member  $BC$  does not change length:



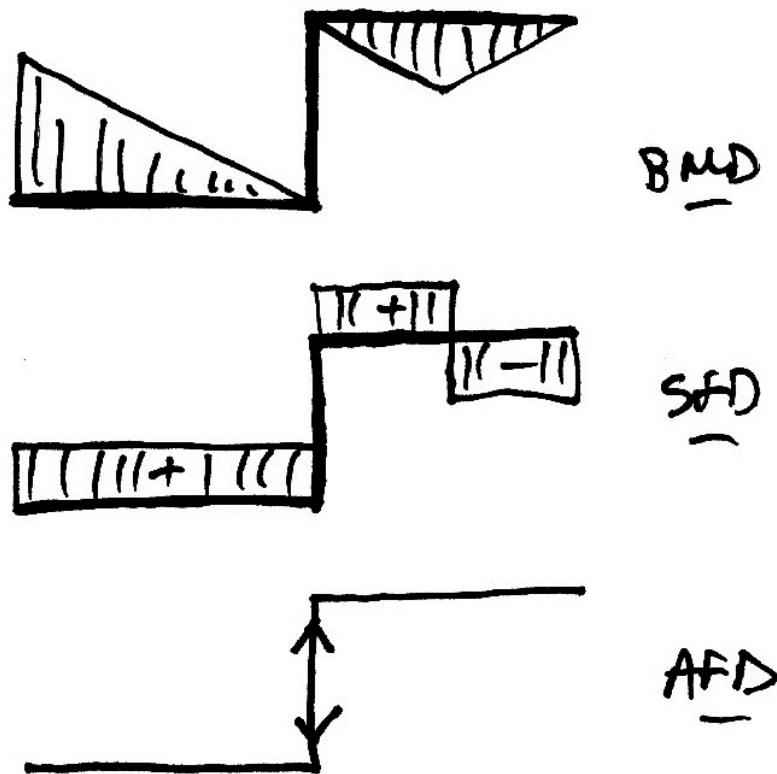
To obtain the overall deflected shape, add the above to that of the beam  $CD$  to get:





The diagram emphasises the point that the horizontal movement at  $C$  and  $D$  must be equal since the beam  $CD$  does not change length.

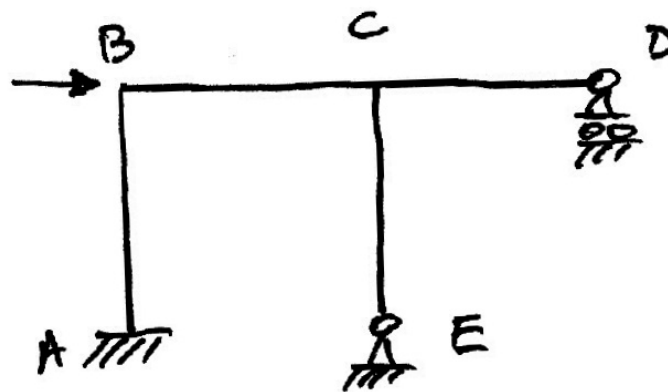
With the reactions and deflected shape established, the remaining diagrams follow easily using the techniques previously described:



### 4.2.14 Example 11

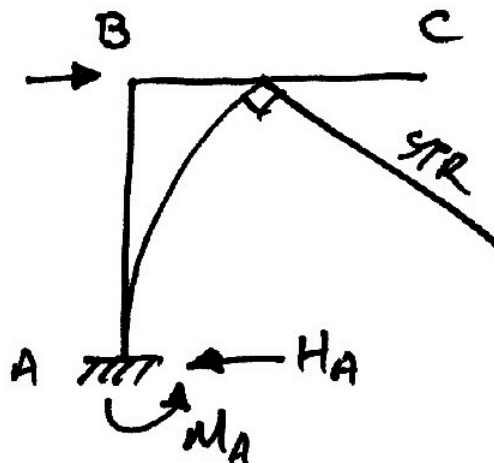
#### Problem

Analyse the following frame for the reactions, bending moment, shear, and axial force diagrams, and draw the deflected shape.

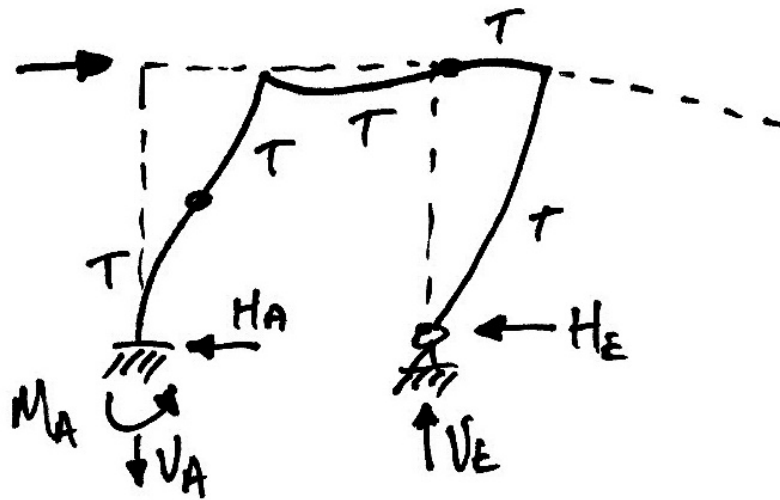


#### Solution

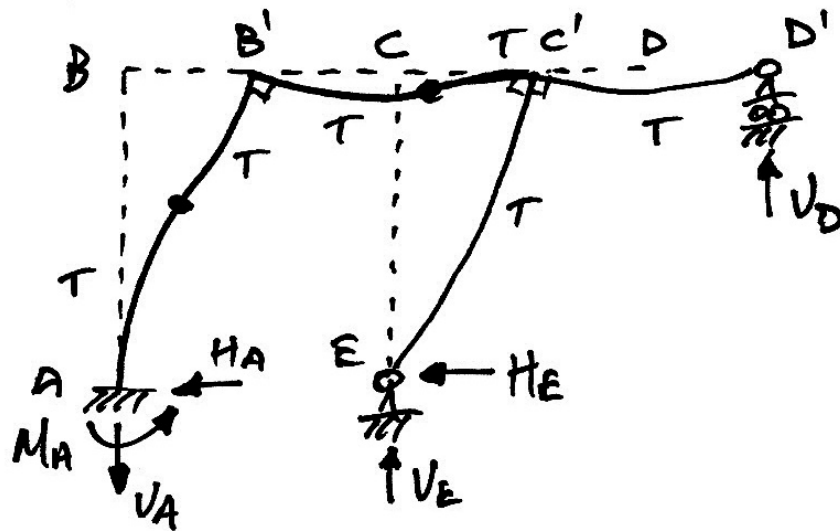
This is a more complex frame than previous frames, and so we will begin by cutting the structure back and gradually adding in the extra members. This is a bigger-scale removal of restraints method, where the members are considered as a type of restraint. We start with the portion ABC, which has been studied previously:



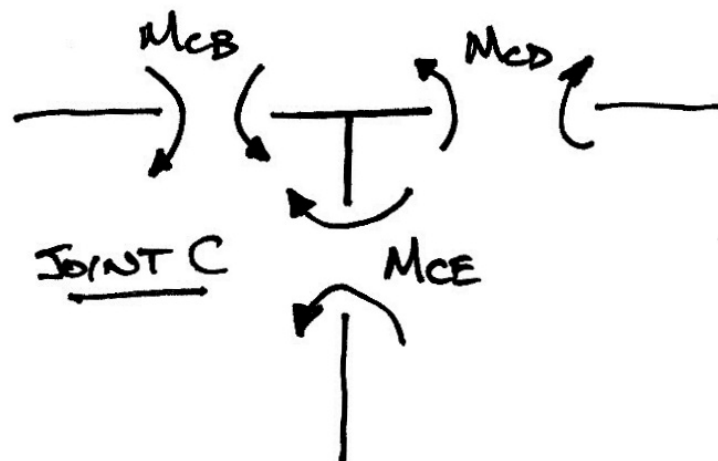
If we now introduce member  $CE$ , we can see that it must push upwards on joint  $C$  to keep  $C$  at the horizontal level it should be at (since member  $CE$  doesn't change length). This tells us that we have an upwards vertical reaction at  $E$ . And since  $\sum F_y = 0$ , we therefore know that  $V_A$  is downwards. Also there must be a horizontal reaction at  $E$  to keep  $E$  from moving right. This causes tension on the outside of member  $CE$ . All this is summarized in the following diagram.



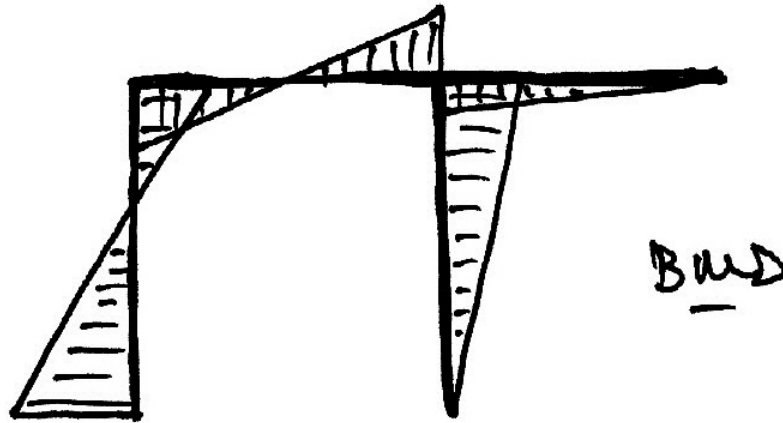
Notice that we have dotted in where member  $CD$  would be if it were connected. This tells us that the vertical reaction at  $D$  must be upwards as follows:



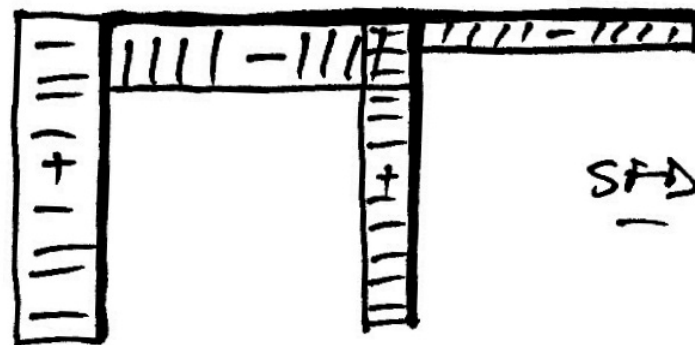
Since we know the sides of the members upon which there is tension, we can assess the equilibrium of joint C:

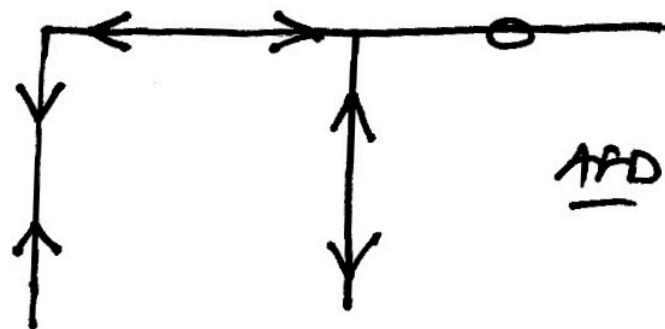


From this, we see that the bending moment in member  $CE$  is biggest. With this information, and the simple force  $\times$  distance strategy of earlier examples, we get:



The shear force and axial force diagrams follow similarly by considering the forces along or transverse to each member. One particularly notable point is that the applied horizontal load ‘splits’ at  $B$ : some goes through shear down to  $A$  (giving  $H_A$ ), whilst the rest (probably smaller) puts member  $BC$  into compression before travelling down member  $CE$  in shear to give  $H_E$ . Thus we have:





## 4.3 Problems

### 4.3.1 Introduction

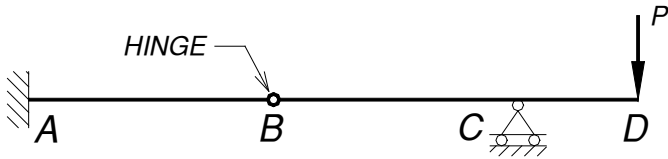

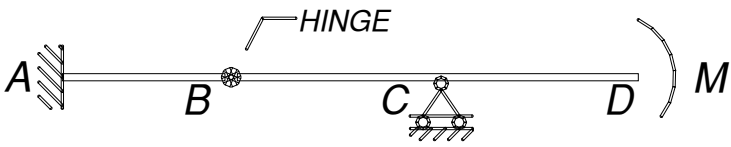
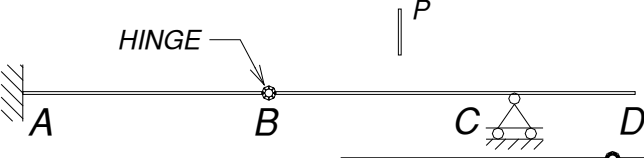
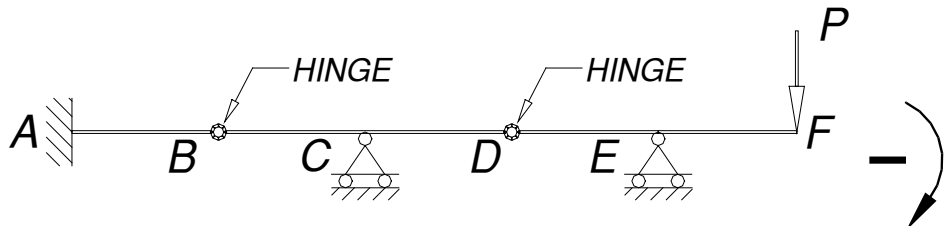
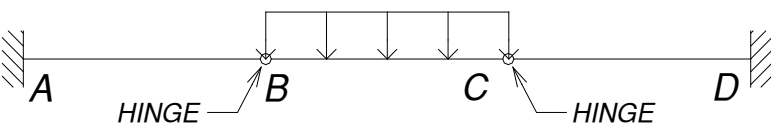
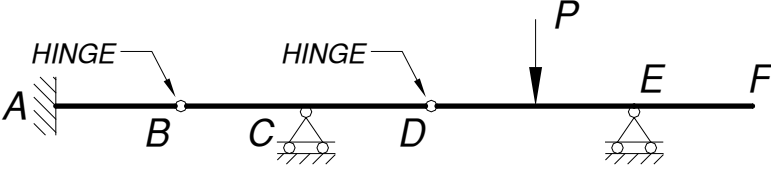
There is no better way to learn qualitative analysis than by practice. So here follows a good variety of determinate and indeterminate structures for analysis.

For each of the following structures, determine the:

- Reactions;
- Bending moment diagram;
- Shear force diagram;
- Axial force diagram;
- Deflected shape.

For the trusses, identify the sense of the force, if any, in each member.

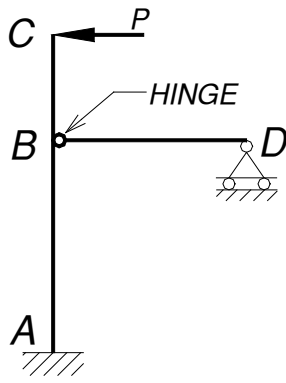
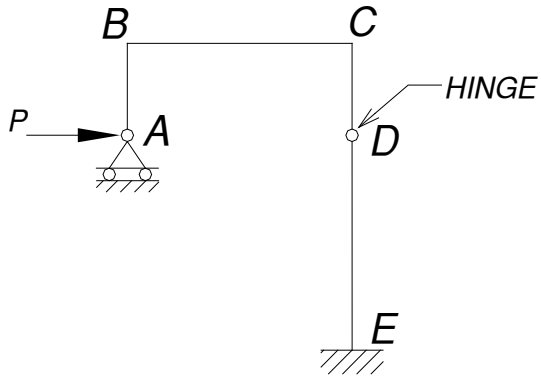
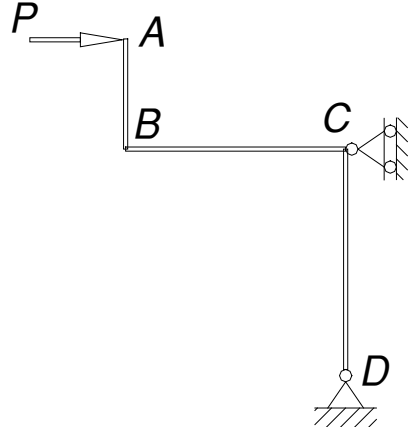
### 4.3.2 Statically Determinate Beams

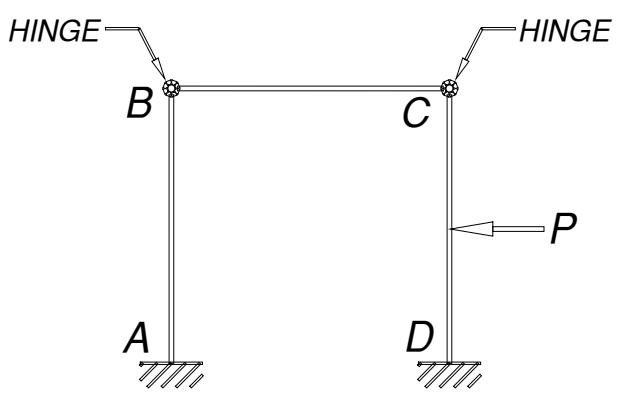
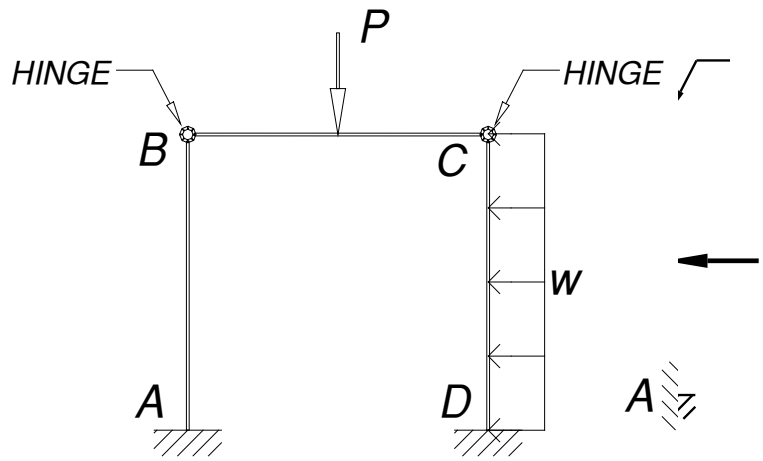
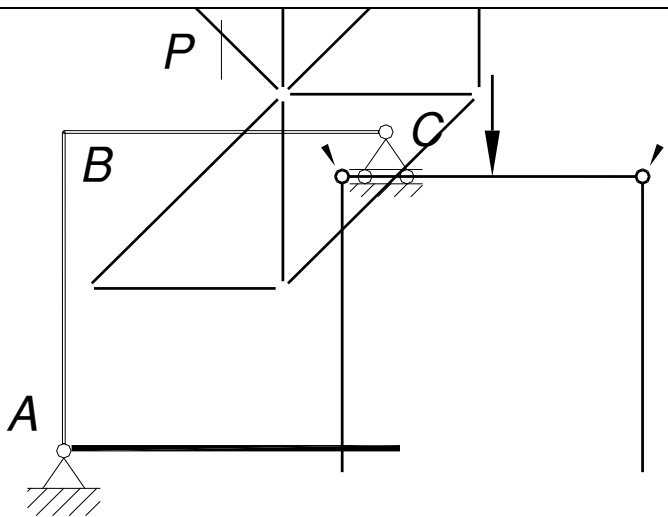
1	
2	
3	
4	
5	
6	
7	

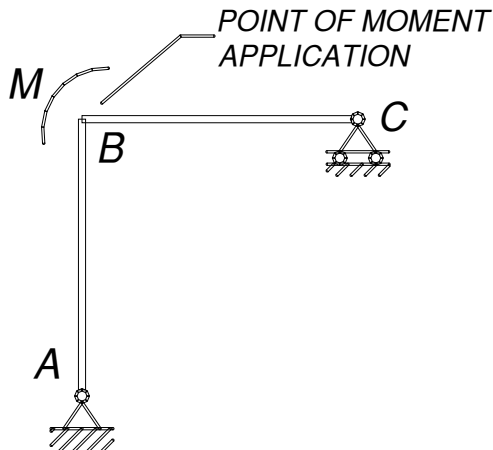
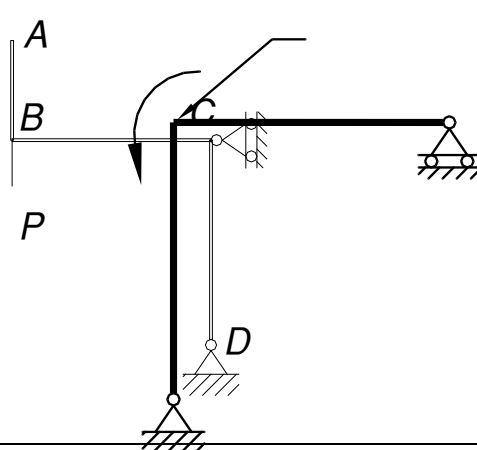
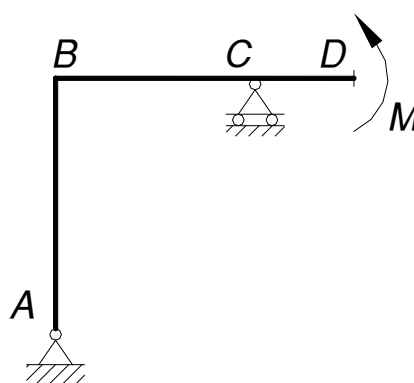


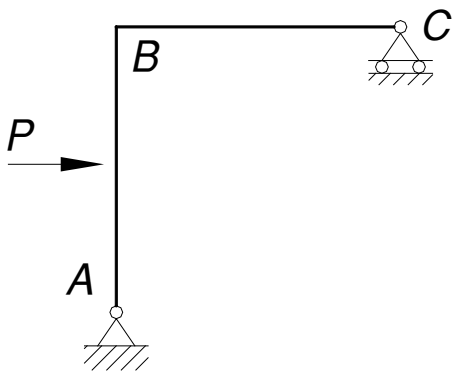
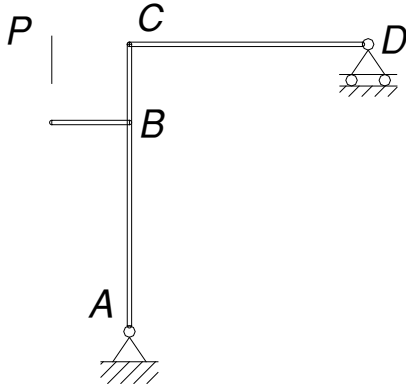
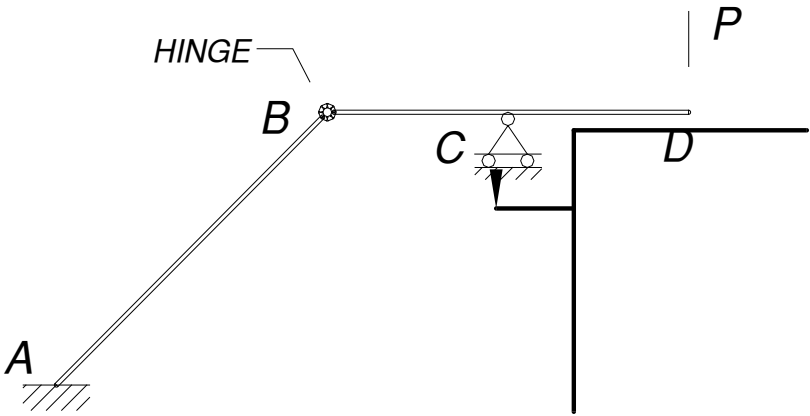
8	
9	
10	
11	
12	
13	
14	

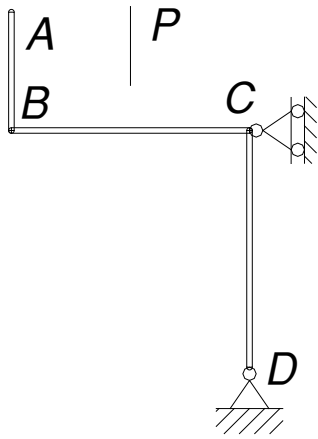
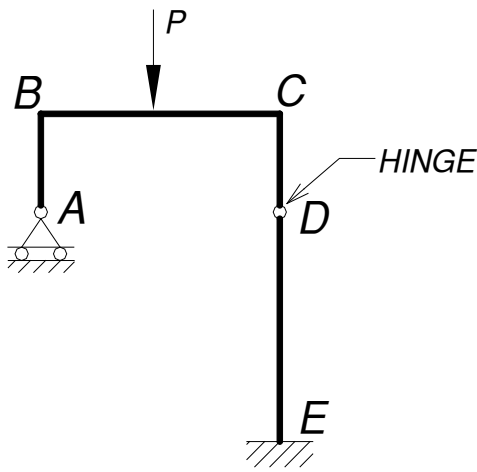
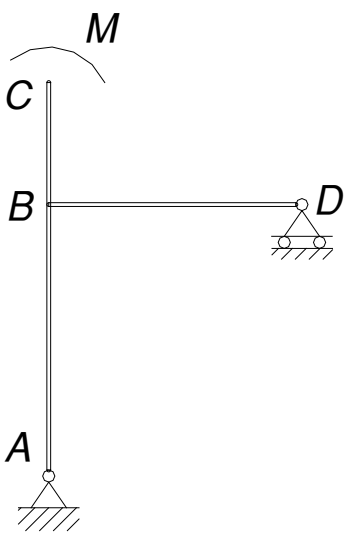
4.3.3 Statically Determinate Frames

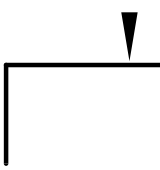
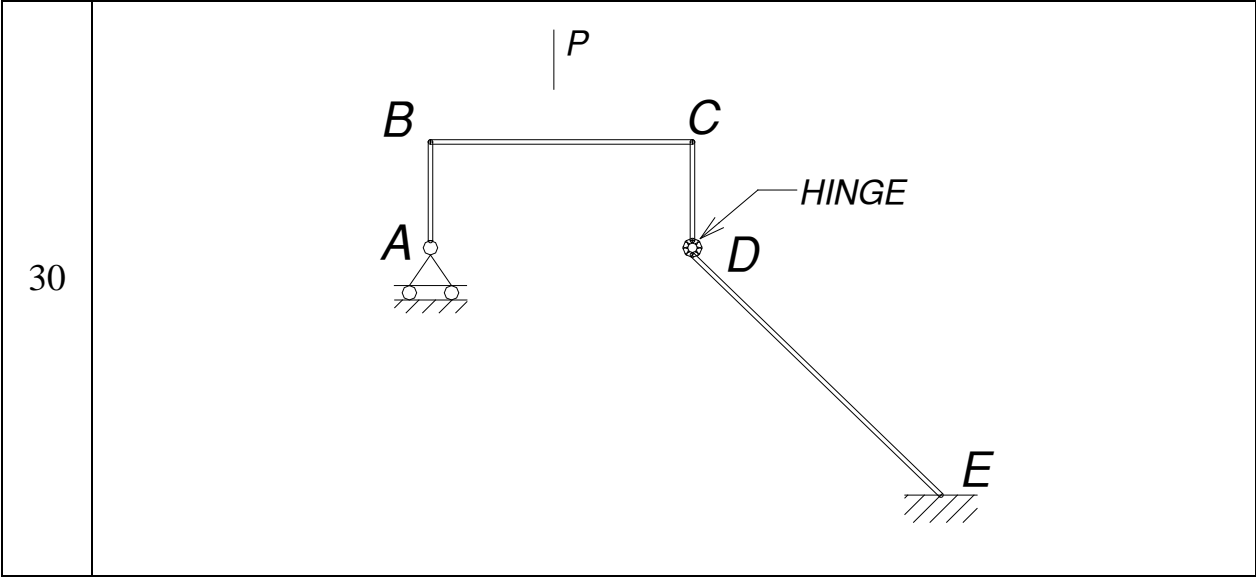
15	 <p>Diagram of a frame structure. It has a fixed support at A. A vertical member AB goes up to B. A horizontal member BC goes right to C. A horizontal member CD goes right to D. A horizontal force <math>P</math> is applied at C to the left. A hinge is located at B. A roller support is at D.</p>
16	 <p>Diagram of a frame structure. It has a fixed support at A. A vertical member AB goes up to B. A horizontal member BC goes right to C. A vertical member CE goes down to E. A horizontal force <math>P</math> is applied at A to the right. A hinge is located at D. A roller support is at E.</p>
17	 <p>Diagram of a frame structure. It has a fixed support at D. A vertical member CD goes up to C. A horizontal member CB goes left to B. A vertical member BA goes up to A. A horizontal force <math>P</math> is applied at A to the right. A roller support is at C.</p>

18	
19	
20	

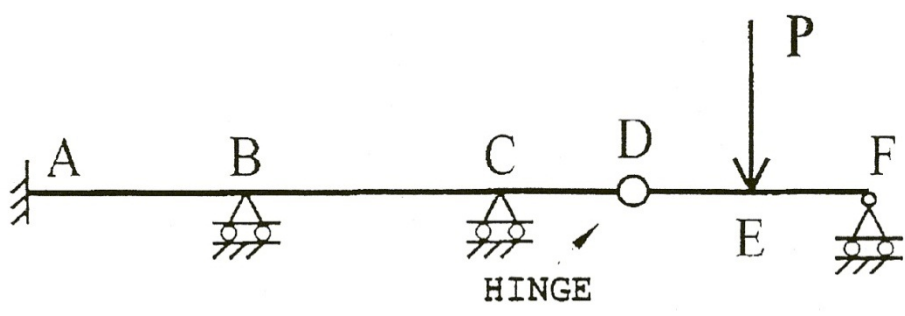
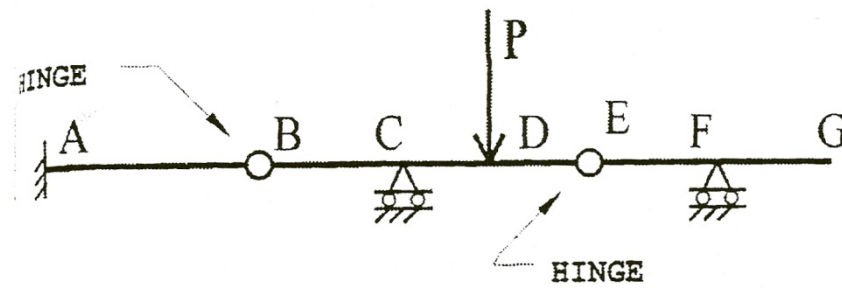
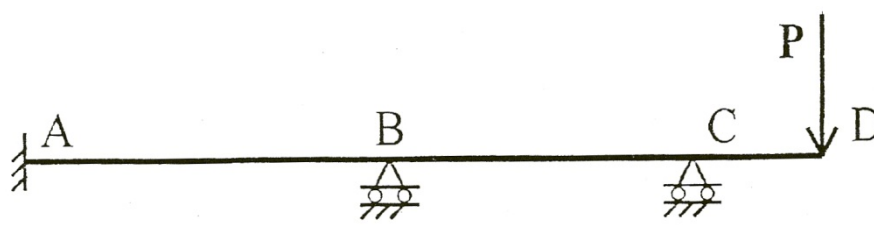
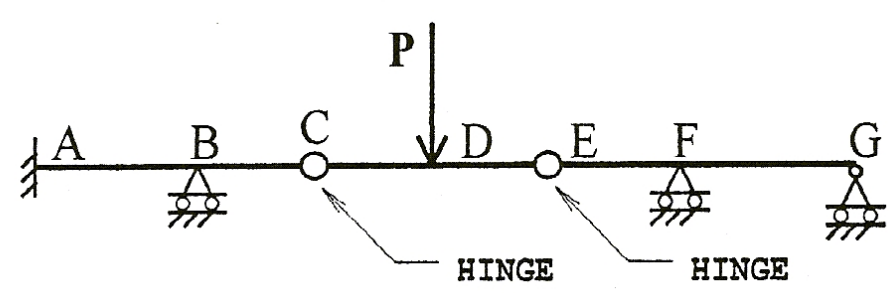
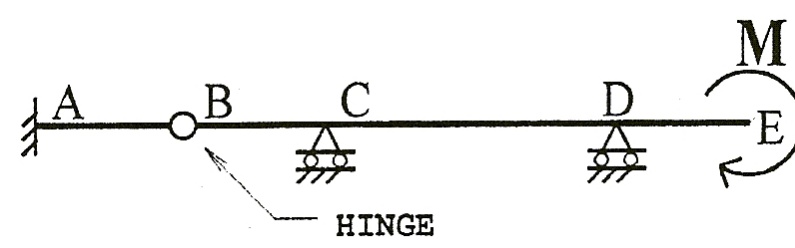
21	 <p>Diagram of a frame structure. It consists of a vertical member AB and a horizontal member BC. Joint A is a pin support. Joint B is the corner joint. A counter-clockwise moment <math>M</math> is applied at joint B. Joint C is a roller support. A label "POINT OF MOMENT APPLICATION" points to joint B.</p>
22	 <p>Diagram of a frame structure. It consists of a vertical member CD and a horizontal member BC. Joint D is a fixed support. Joint C is the corner joint. A horizontal force <math>P</math> is applied at joint B, acting to the left. Joint A is a fixed support. Joint C is a roller support.</p>
23	 <p>Diagram of a frame structure. It consists of a vertical member AB and a horizontal member BC. Joint A is a pin support. Joint B is the corner joint. Joint C is a roller support. Joint D is the end of the horizontal member. A clockwise moment <math>M</math> is applied at joint D.</p>

24	 <p>A frame structure with a pin support at point A. A horizontal force <math>P</math> is applied at point B, pointing to the right. The structure consists of a vertical member AB and a horizontal member BC. Point C is supported by a roller.</p>
25	 <p>A frame structure with a pin support at point A. A horizontal force <math>P</math> is applied at point B, pointing to the left. The structure consists of a vertical member AB and a horizontal member BC. Point C is connected to point D by a horizontal member, and point D is supported by a roller.</p>
26	 <p>A frame structure with a pin support at point A. A hinge is located at point B. The structure consists of a diagonal member AB, a horizontal member BC, and a horizontal member CD. Point C is supported by a roller. A horizontal force <math>P</math> is applied at point D, pointing to the right.</p>

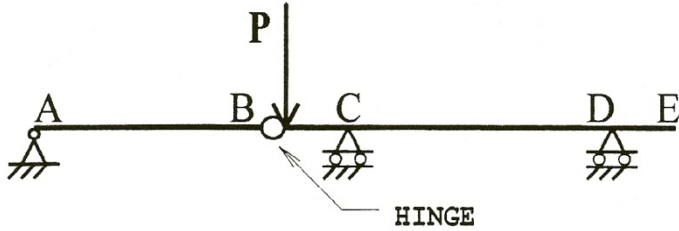
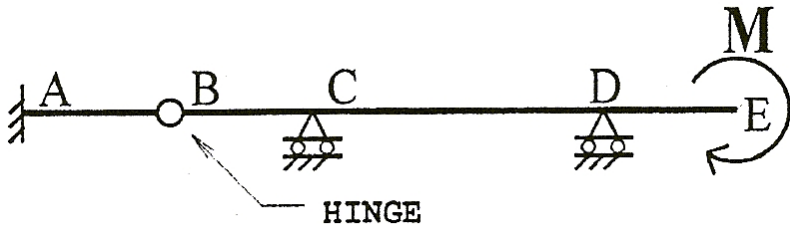
27	 <p>Diagram of a frame structure. It consists of a vertical member AB, a horizontal member BC, and a vertical member CD. A point load <math>P</math> is applied at joint B. Support C is a roller support, and support D is a pin support.</p>
28	 <p>Diagram of a frame structure. It consists of a pin support at A, a horizontal member BC, and a vertical member CE. A point load <math>P</math> is applied at the midpoint of BC. A hinge is located at joint D on member CE.</p>
29	 <p>Diagram of a frame structure. It consists of a pin support at A, a vertical member AB, a horizontal member BD, and a vertical member DC. A clockwise moment <math>M</math> is applied at joint C.</p>



**4.3.4 Statically Indeterminate Beams**

31	
32	
33	
34	
35	

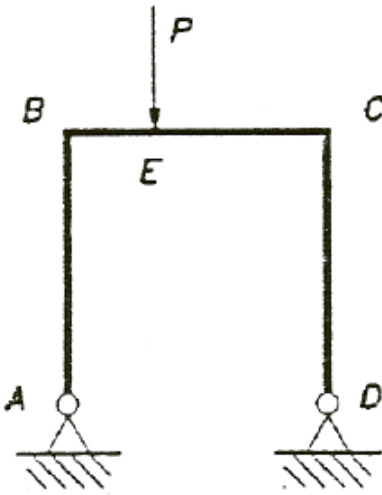
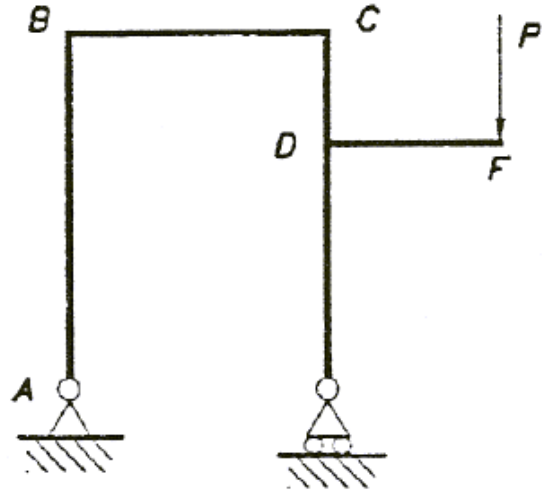
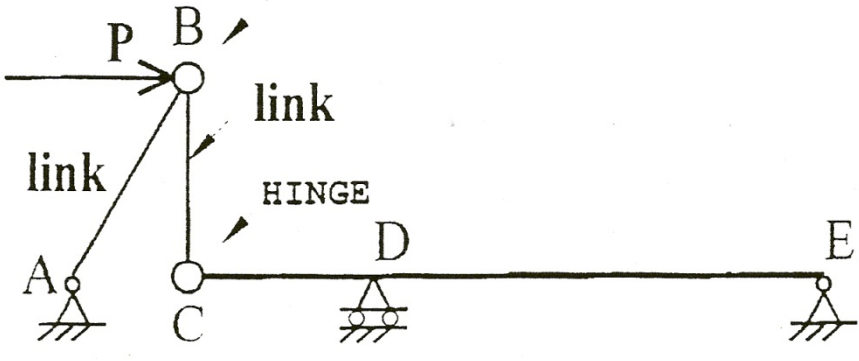


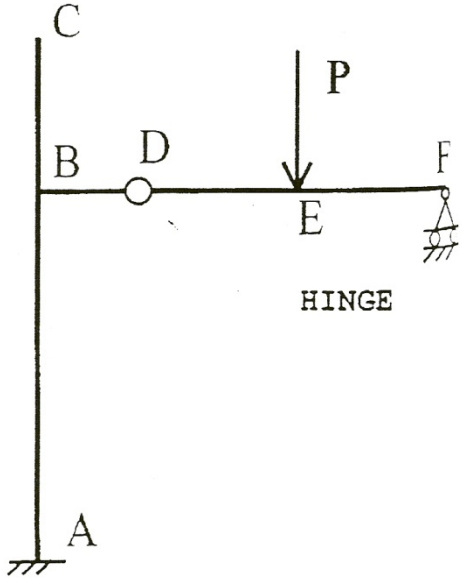
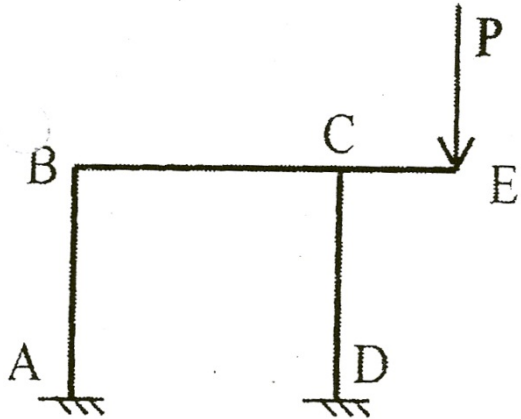
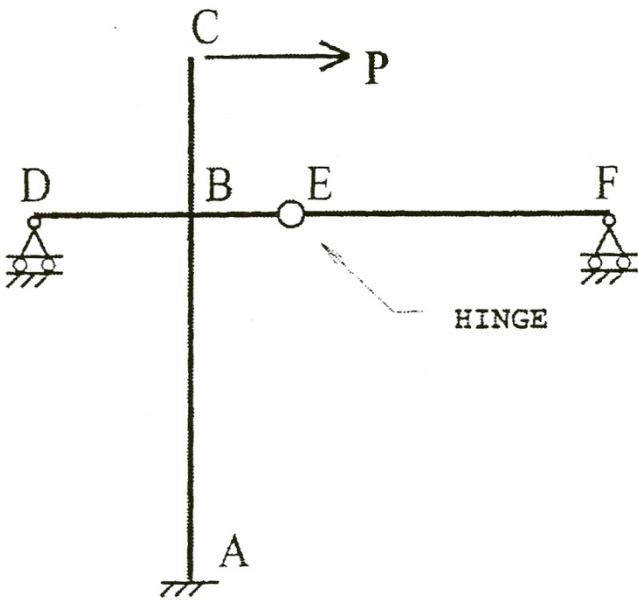
36	 <p>The diagram shows a horizontal beam with points A, B, C, D, and E marked along its length. At point A, there is a pin support. At point B, there is a hinge support, indicated by a circle and a label 'HINGE' with an arrow pointing to it. A vertical downward point load 'P' is applied at point B. At point C, there is a roller support. At point D, there is another roller support. The beam ends at point E.</p>
37	 <p>The diagram shows a horizontal beam with points A, B, C, D, and E marked along its length. At point A, there is a fixed support. At point B, there is a hinge support, indicated by a circle and a label 'HINGE' with an arrow pointing to it. At point C, there is a roller support. At point D, there is another roller support. At point E, there is a clockwise moment 'M' applied, represented by a curved arrow. The beam ends at point E.</p>

4.3.5 Statically Indeterminate Frames

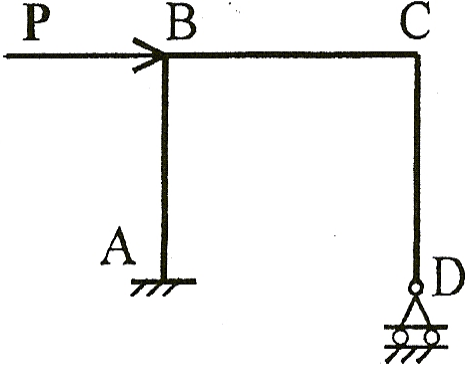
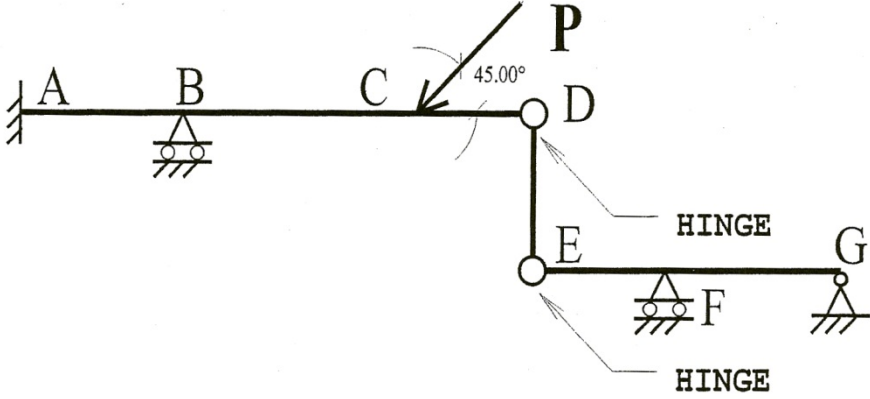
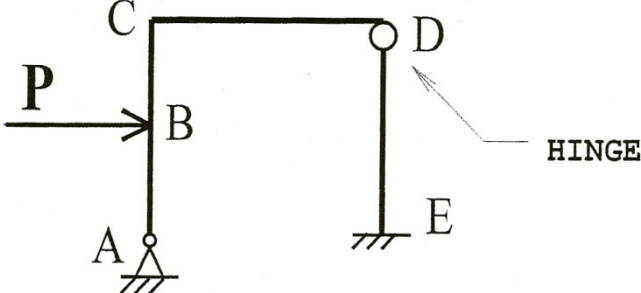
38	
39	
40	
41	

42	
43	
44	
45	

46	 <p>A portal frame structure with pin supports at joints A and D. The frame consists of vertical members AB and CD, and a horizontal beam BC. A vertical load <math>P</math> is applied downwards at point E, which is the midpoint of beam BC.</p>
47	 <p>A portal frame structure with pin supports at joints A and D. The frame consists of vertical members AB and CD, and a horizontal beam BC. A horizontal member DF is attached to joint D, with a vertical load <math>P</math> applied downwards at its free end F.</p>
48	 <p>A frame structure with a pin support at joint A, a hinge at joint D, and a roller support at joint E. The structure includes a diagonal link AB, a vertical link BC, and a horizontal beam CE. A horizontal load <math>P</math> is applied to the right at joint B. The beam CE is divided into segments CD and DE.</p>

49	 <p>Diagram 49: A frame structure. A vertical member AB is fixed at support A. A horizontal member BE is attached to B. A vertical load <math>P</math> acts downwards at point E. A hinge is located at point D on the horizontal member BE. A roller support is located at point F. Points C, B, D, E, and F are labeled. The word "HINGE" is written below the structure.</p>
50	 <p>Diagram 50: A frame structure. Two vertical members AB and CD are fixed at supports A and D respectively. They are connected by a horizontal member BC. A vertical load <math>P</math> acts downwards at point E, which is the end of a horizontal extension CE from point C.</p>
51	 <p>Diagram 51: A frame structure. A vertical member AB is fixed at support A. A horizontal member DF is attached to B. A horizontal load <math>P</math> acts to the right at point C, which is the top of member AB. A hinge is located at point E on the horizontal member DF. Roller supports are located at points D and F. The word "HINGE" is written below the structure.</p>

52	<p>Diagram of a frame structure. It consists of a vertical member AB fixed at support A, and a horizontal member BE. A hinge is located at joint C on member BE. A downward point load P is applied at joint D on member BE. Support E is a roller support. A dashed arrow points to joint C with the label "HINGE".</p>
53	<p>Diagram of a frame structure. It consists of a vertical member AB fixed at support A, a horizontal member BC, and a vertical member CE fixed at support E. A downward point load P is applied at joint D on member BC.</p>
54	<p>Diagram of a frame structure. It consists of a member AB fixed at support A, a horizontal member CD, and an inclined member BC. A downward point load P is applied at joint C. Support D is a roller support.</p>

55	
56	
57	

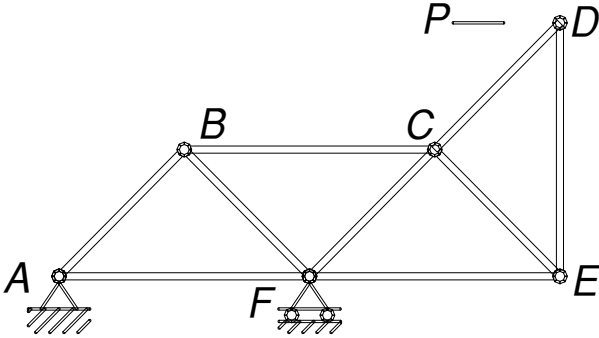
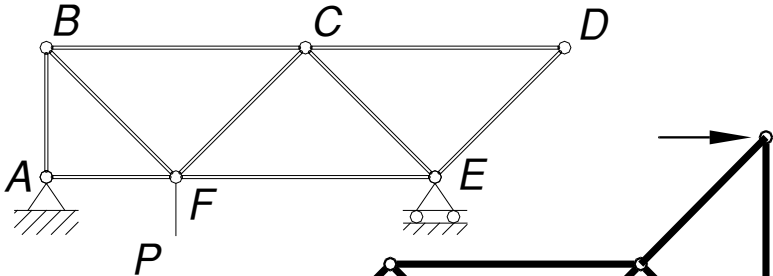
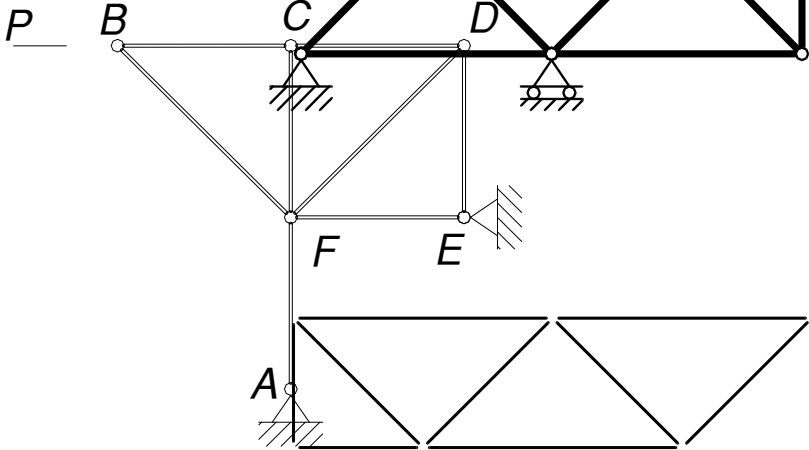
4.3.6 Trusses

58	
59	
60	
61	



62	
63	
64	
65	

66	
67	
68	
69	

70	
71	
72	
73	